Graph Searching Games with Multiple Robbers and Games with Imperfect Information

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Complexity Measures

What is a simple graph?

- a small graph
- can be described by certain formalisms
  - in certain logics (e.g., FO, LTL)
  - by graph grammar
  - by an inductive construction (e.g., clique-decomposition)
  - algebraically (e.g., (bi-)rank-width)
  - ...
- with few intertwined cycles
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Cyclicity Measures

- tree-width and path-width [Robertson, Seymour]
- directed tree-width [Johnson, Robertson, Seymour, Thomas]
- DAG-width [Berwanger, Dawar, Hunter, Kreutzer; Obdržálek]
- directed path-width [Reed, Seymour, Thomas]
- Kelly-width [Hunter, Kreutzer]
- entanglement [Berwanger, Grädel]
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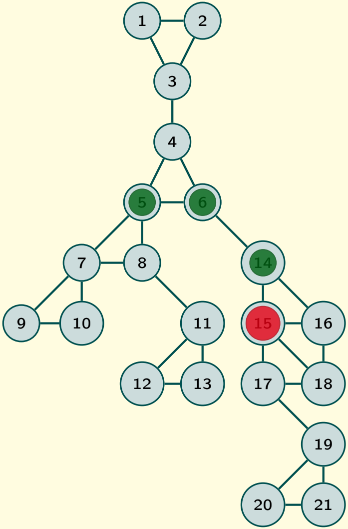
Tree-width: Game Theoretial Definition



Game rules:

- k Cops, one Robber
- Robber runs along cop free paths
- Cops fly
- Cops want to capture Robber

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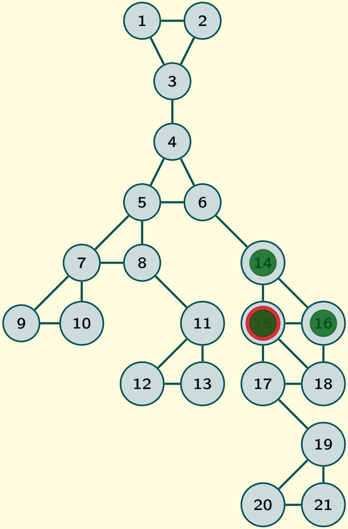

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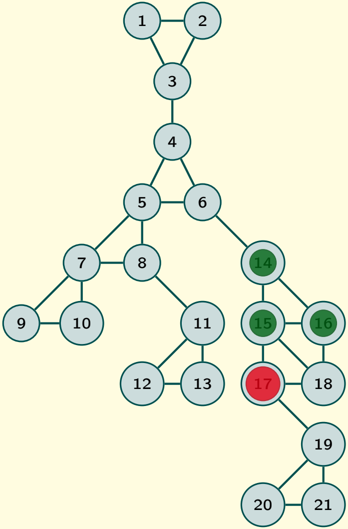

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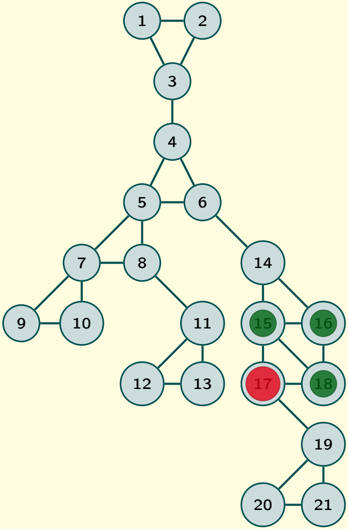

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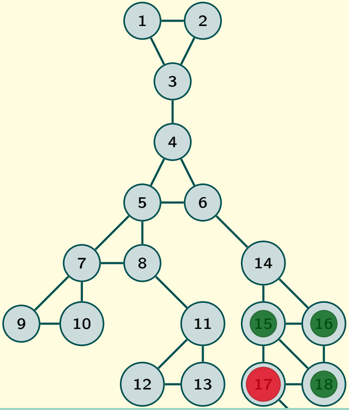

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Tree-width = minimal number cops monotonously capturing Robber − 1

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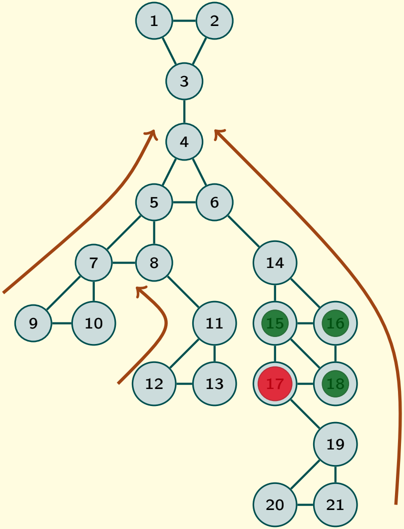

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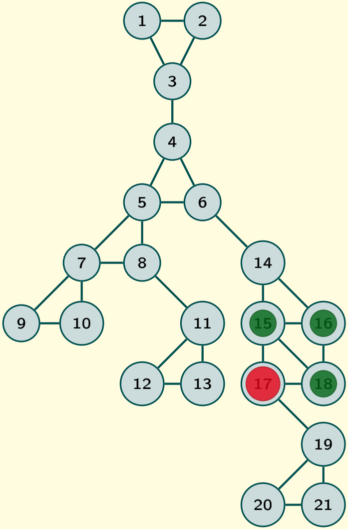

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Path-width:
the same,
but robber invisible.

DAG-width

- straightforward generalisation to directed graphs
  (= tree-width on undirected)
- the robber can run only along directed paths

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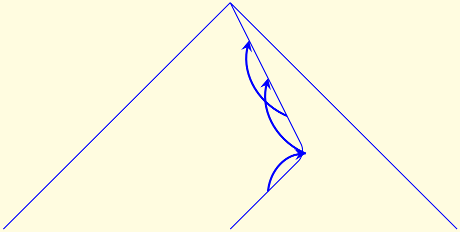
Compare to other generalisations:

- Kelly-width: the robber is invisible and inert
- directed tree-width: the robber locked in his component
- bounded DAG-width: only parity games

Bounded and Unbounded DAG-width

Bounded: only few paths in one direction

- DAGs (1)
- trees with short back-edges



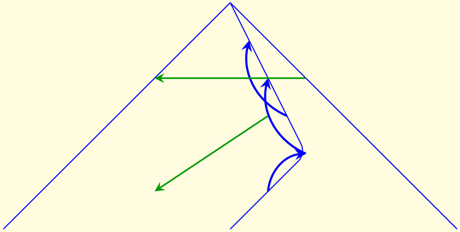
Unbounded:

- Grids
- trees with many long back-edges

Bounded and Unbounded DAG-width

Bounded: only few paths in one direction

- DAGs (1)
- trees with short back-edges and horizontal connections



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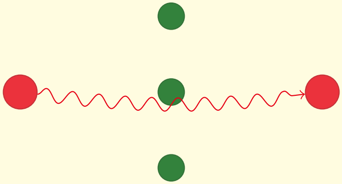
Generalisation to Multiple Robbers

Same rules, but multiple robbers:

- all r robbers must be captured
- robbers can jump to each other
- global monotonicity: no robber can access a vertex that was inaccessible to all robbers



Main Result

Theorem

one robber captured by k cops

$$\Rightarrow$$

r robbers captured by $k \cdot r$ cops

Rough proof idea:

- a team of k cops against each robber
- play independently with each team

1. This works for tree-width...
2. This needs improvement for directed case...

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Stronger assumption, stronger conclusion: more paths for robber(s)

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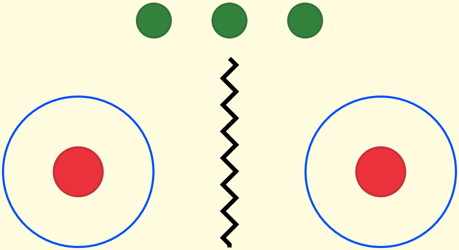

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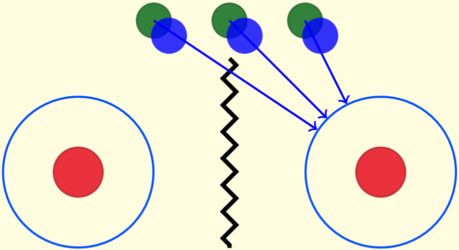

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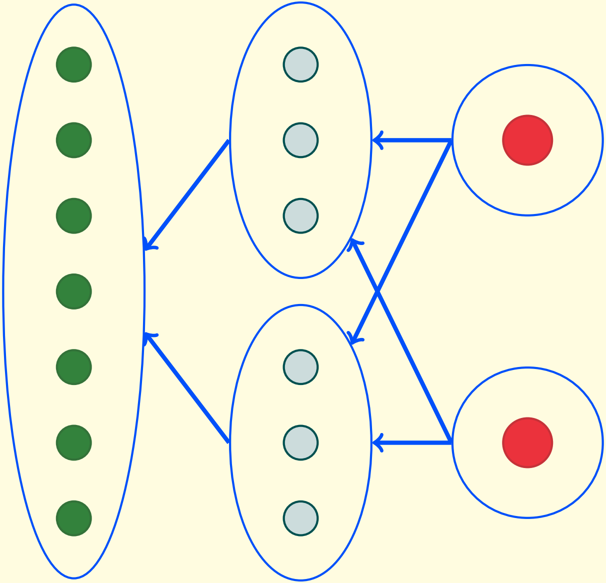

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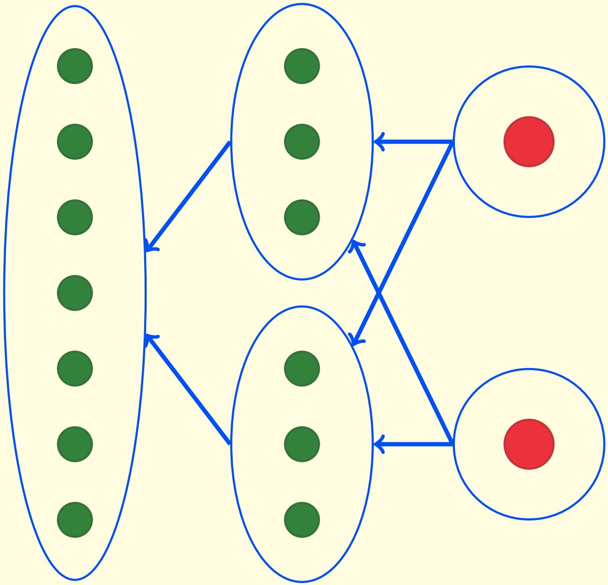

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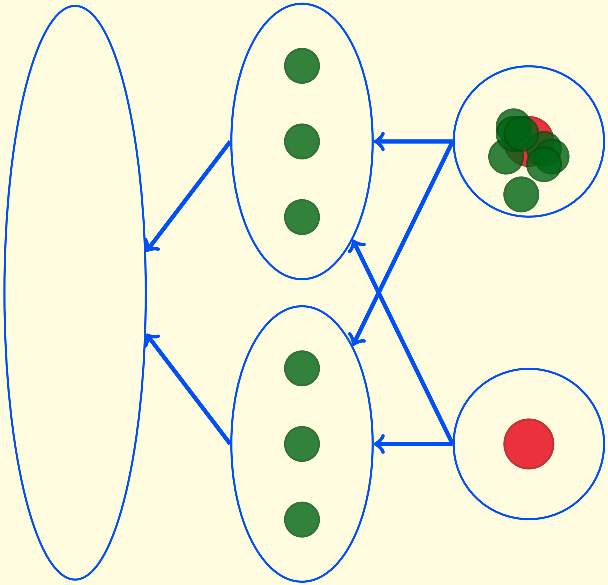

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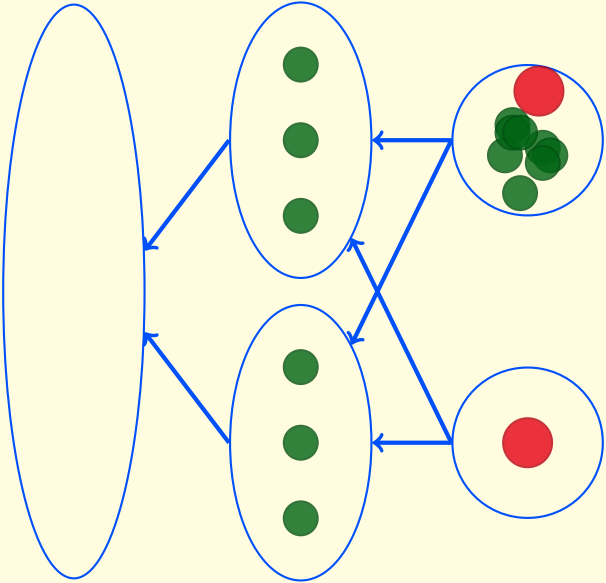
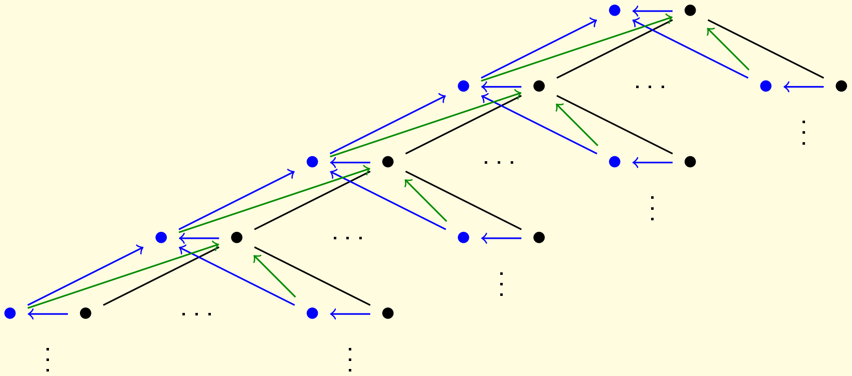
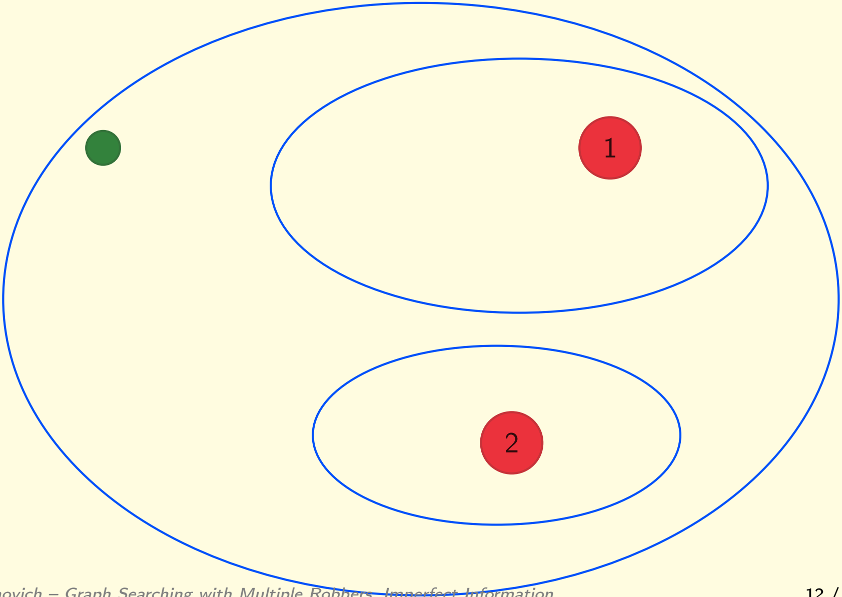


Offhanded Cops

Never place a cop outside robber's component.



Solving Monotonicity Problem for r Robbers



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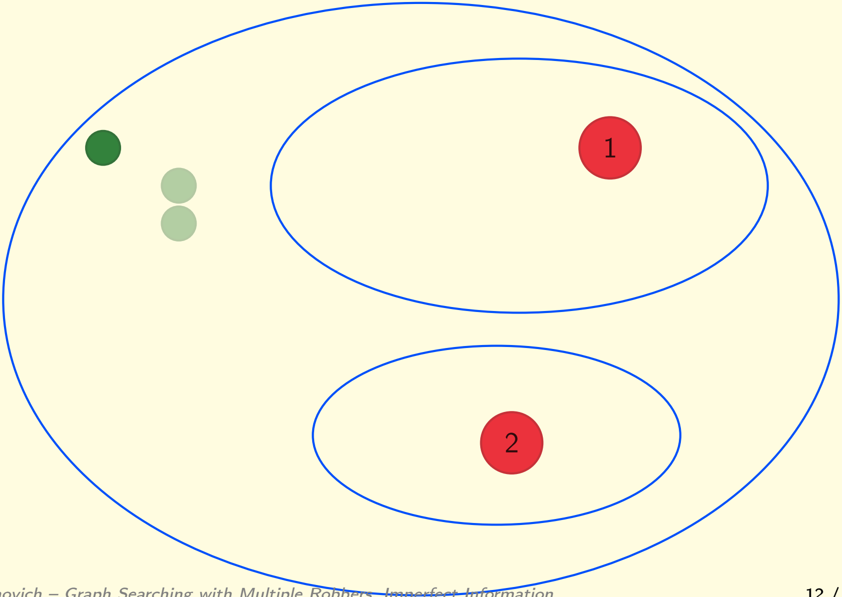

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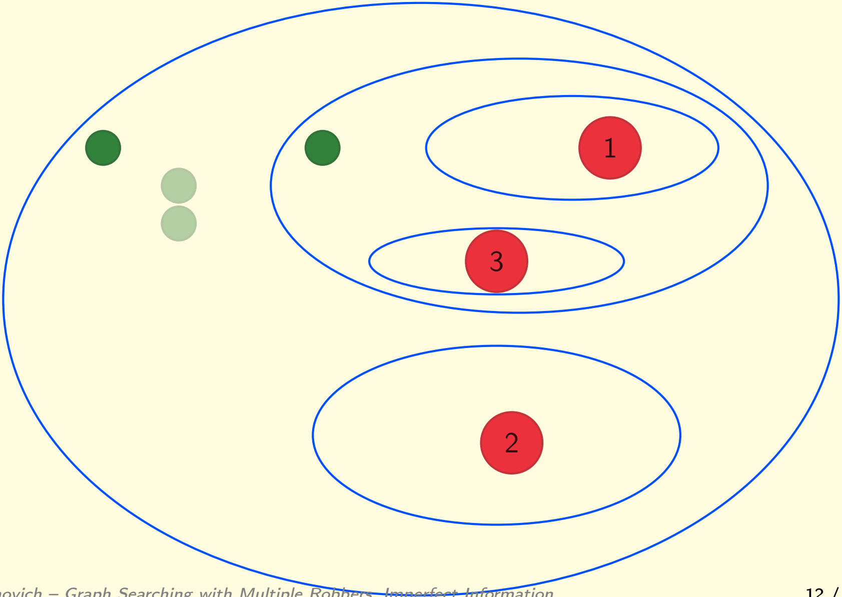

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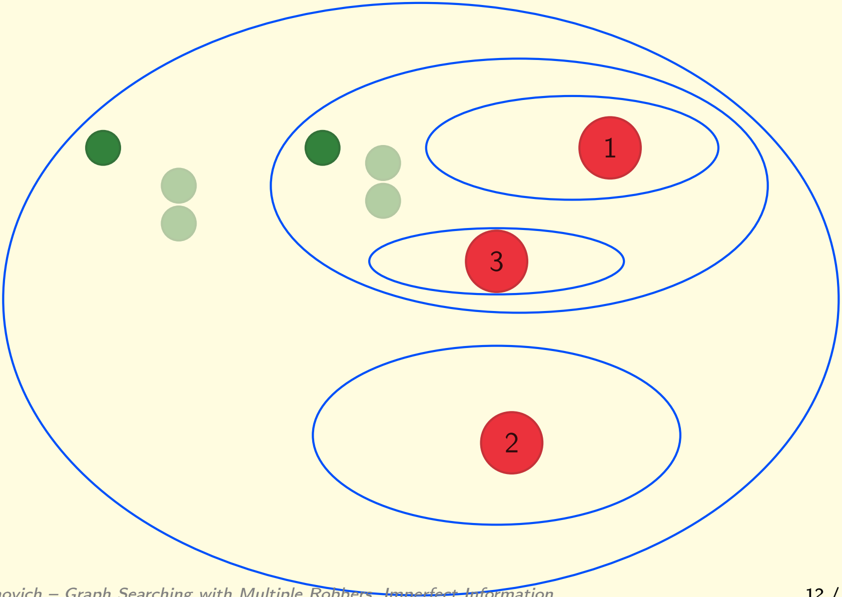

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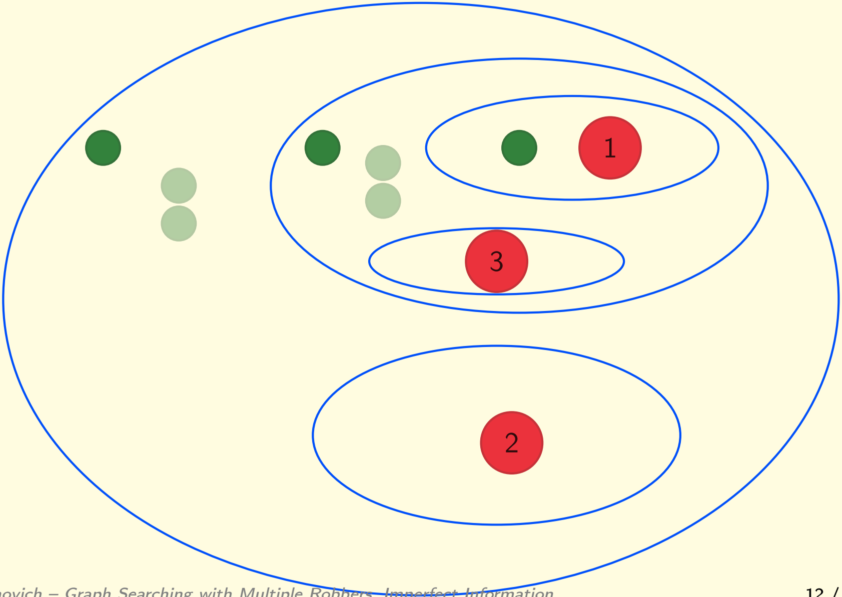

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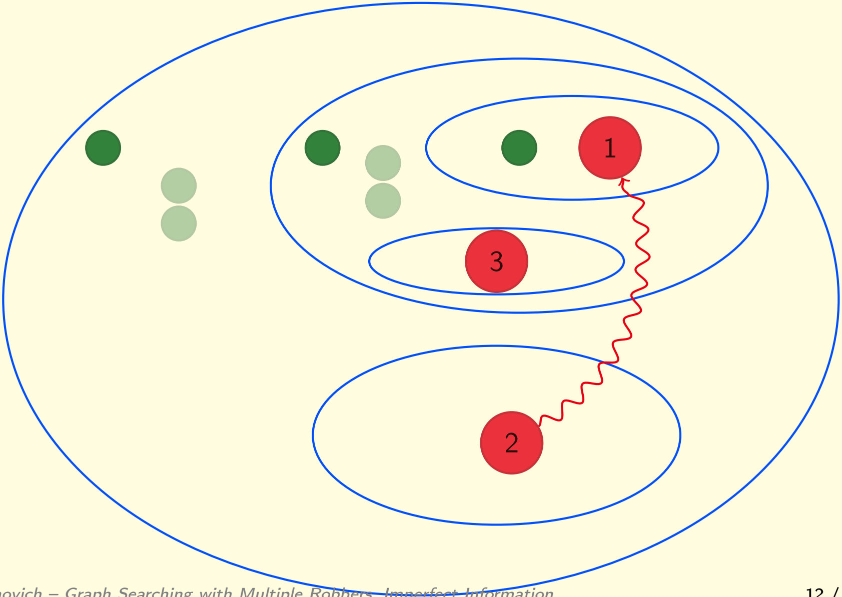

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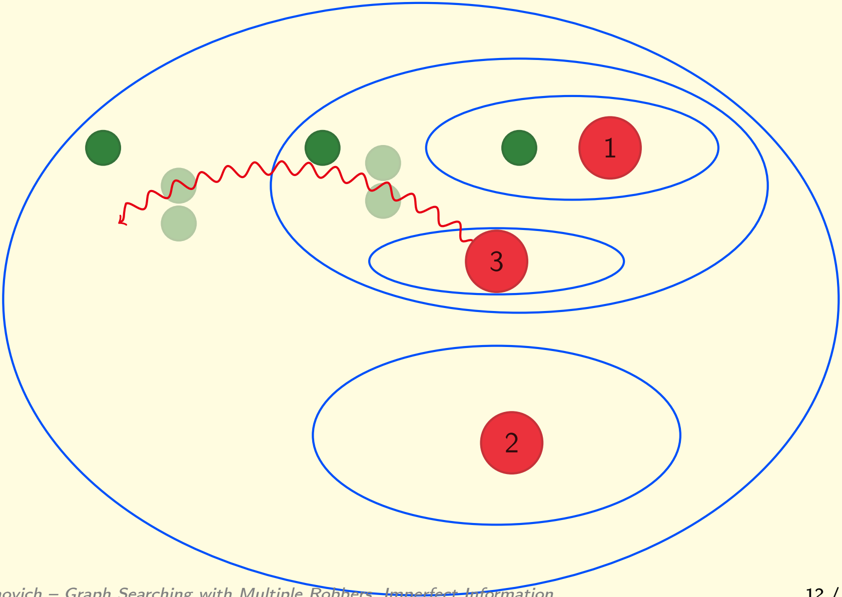

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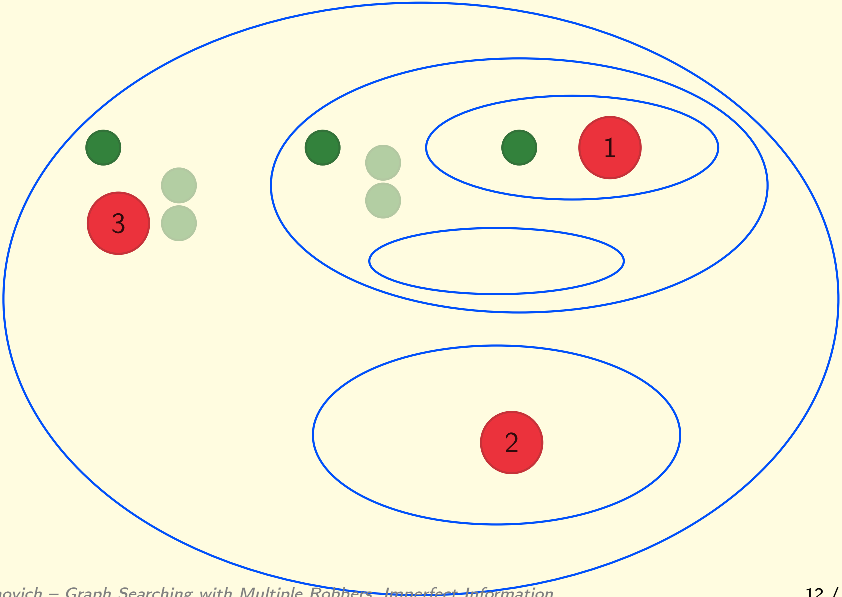
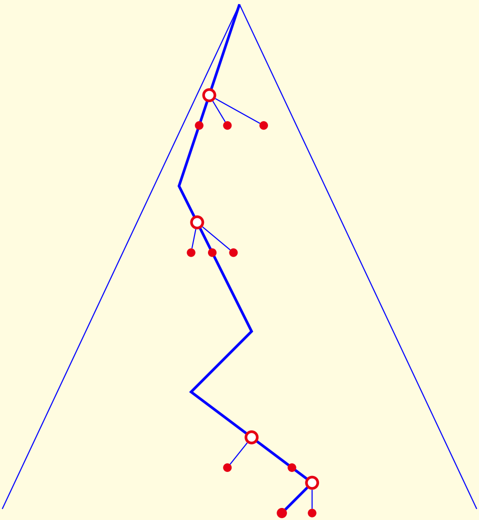


Maintaining and linearizing several plays



Corollaries

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Parity games of bounded imperfect information on graphs of bounded DAG-width can be solved in polynomial time.

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Theorem (Robbers Hierarchy)

There are classes of graphs on which every new robber demands new cops until directed path-width is reached.