A Perfect-Information Construction for Coordination in Games*

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Abstract

We present a general construction for eliminating imperfect information from games with several players who coordinate against nature, and to transform them into two-player games with perfect information while preserving winning strategy profiles. The construction yields an infinite game tree with epistemic models associated to nodes. To obtain a more succinct representation, we define an abstraction based on homomorphic equivalence, which we prove to be sound for games with observable winning conditions. The abstraction generates finite game graphs in several relevant cases, and leads to a new semi-decision procedure for multi-player games with imperfect information.

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1 Introduction

A game with perfect information is one where a player knows the state of the play at any stage. If he does not, we speak of a game with imperfect information. Analysing games with perfect information appears conceptually easier than those of imperfect information, which require handling the uncertainty of players. We present a generic construction for eliminating imperfect information from games where players coordinate against nature, and transform them into games with perfect information while preserving winning strategies.

We consider infinite games played on finite graphs [10, 16]. Plays proceed in stages in which a token is moved along the edges, forming an infinite path. A state corresponds to the node of the graph holding the token. Under perfect information, the current state is explicitly announced to each player at every stage. Under imperfect information, the announcement is made with uncertainty modelled by an indistinguishability relation between states.

In our setting, there are n players that form a coalition against nature; at each stage, the players choose simultaneously an action and nature moves the token along an edge compatible with these choices. The objective of the players is to ensure that the outcoming path satisfies a given winning condition, regardless of the moves of nature. We focus on the *coordinated winning strategy problem*: to decide whether the grand coalition has a joint strategy to ensure a win, and to construct one, if this is the case. When we speak about the solution of a game throughout the paper, we mean the solution to both the decision and the construction variant of the coordinated winning strategy problem. These problems are central to the area of distributed controller synthesis (see [8, 11, 5]).

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For the case of a single player against nature, the winning strategy problem has been formulated and solved by Reif 25 years ago [15] – this basic case does not raise the issue of coordination. Reif's approach proceeds by elimination of imperfect information, as the author phrases it: for a given game G with imperfect information, a game G^+ with perfect information is constructed in a way that resembles the powerset construction for determinising finite automata. The states of the perfect-information game G^+ correspond to subsets of states in the imperfect-information game G. Intuitively, any set Π of plays in G that are indistinguishable for the player corresponds to one play π in G^+, and the subset state reached in G^+ via π consists of the states reachable in G when one of the plays from Π is played. The states of G^+ thus represent enough of the player's knowledge about the current state of a play in G to allow transferring his strategies from G^+ to G in a way that preserves winning.

Reif's subset construction allows to reduce (both the decision and the construction variant of) the winning strategy problem for a game with imperfect information played by a player against nature to the corresponding problem in a two-player game of perfect information over a state space that may be exponentially larger. Although the original procedure addressed only games with simple, reachability winning conditions, it extends easily to general observable ω-regular conditions and the resulting games belong to the class of infinite games on finite graphs that is well understood. They are determined with simple strategies (of bounded memory) and they can be solved algorithmically: it is decidable whether a player has winning strategies, and if so, one can construct one (see, e.g., [6]). Thus, the reduction yields solution procedures for the original games of imperfect information and further insights, e.g., about the memory requirement of winning strategies.

Unfortunately, the classical subset construction is not sound in settings that involve more than one player. In fact, the coordinated winning strategy problem is generally undecidable already for two players against nature [12, 13, 17], which implies not only that the subset construction is inadequate for eliminating imperfect information in games with two or more players, but, moreover, that any procedure that transforms a game with imperfect information over a finite graph into one with perfect information over a possibly larger but still finite graph will fail to preserve the solution to the winning strategy problem in the general case. In [1], Arnold and Walukiewicz give a concrete example of a game with two players against nature where winning strategies depend, at each stage, on the number of previous stages – to keep track of this number, a perfect-information variant of the game would require infinitely many states.

We are interested in constructions that generalise Reif's classical approach in the sense that they transform an n-player game G with imperfect information into a two-player zero-sum game G^+ with perfect information such that

(i) the grand coalition in G has a winning strategy against nature if, and only if, the first player has a winning strategy in G^+;
(ii) winning strategies of the first player in G^+ can be translated uniformly into joint winning strategies of the grand coalition in G and vice versa.

If we think of players as components of a system (each with imperfect information about the global state) that shall follow a joint strategy prescribed by the system designer, such a construction allows to formulate the task of the designer in terms of games between two players: the system designer and nature (or the environment, in the phrasing of the distributed-systems literature). One desirable property of such a construction is that it produces instances of perfect-information games that are finite, for possibly large classes of input games with imperfect information – even if, as pointed out in the previous paragraph, this cannot work for the general case.

Several approaches to identify computationally manageable classes of games with imperfect information among several players have been proposed during the last decade [8, 9, 3, 7, 14, 4]. As a common pattern, tractability is ensured by restricting the way information flows between players.

In this paper, we take a different approach and propose a sufficient, though undecidable, condition for manageability of games with imperfect information. Our perfect-information construction is based on the unravelling of an imperfect-information game as a tree with epistemic models associated to nodes. Intuitively, an epistemic model is a snapshot of what players know at a stage of the game. The unravelling generates a two-player game of perfect information on an infinite tree.

To obtain a more succinct representation, we perform an abstraction by taking the quotient of the tree under homomorphic equivalence of epistemic models. We prove that this abstraction method is sound for imperfect-information games with *observable* ω-regular winning conditions. Consequently, all games that yield a finite quotient admit computable solutions. In particular, this gives an alternative proof for the decidability of games with hierarchical information and observable regular winning conditions. Our proof provides an elementary solution for these games, whereas previous results rely on the simulation theorem of alternation tree automata by nondeterministic ones.

2 Preliminaries

2.1 Distributed Games

We consider games played by n players, $0, 1, \dots, n-1$, against nature. We refer to a list of elements $x = (x_i)_{i<n}$, one for each player, as a *profile*. The *grand coalition* is the set $\{0, \dots, n-1\}$ of all players; nature is not regarded as a player.

Beforehand, we fix a set A_i of *actions* available to Player i, and we denote by A the set of all action profiles. A *distributed game for n players with imperfect information* is described by a structure $\mathcal{G} = (V, \Delta, (\sim_i)_{i<n}, W)$ where V is a finite set of *positions*, $\Delta \subseteq V \times A \times V$ is a *move* relation, and each $\sim_i$ is an equivalence relation on V called the *indistinguishability* relation of Player i. Finally, W is a subset of V^ω describing the *winning condition.*

A *play* in $\mathcal{G}$ is a sequence of positions $\pi = v_0 v_1 v_2 \dots$ such that, for every stage $l \geq 0$, there exists an action profile a_l such that $(v_l, a_l, v_{l+1}) \in \Delta$. We denote the set of all plays by Π. In general, the winning condition is just a set of plays, $W \subseteq \Pi$. We will often focus on ω-regular sets W. More specifically, we will be interested in observable winning conditions. For a set of colours C, we say that a colouring $\Omega : V \to C$ is *observable* if, whenever $\Omega(v) \neq \Omega(w)$, we have $v \not\sim_i w$, for all players i. An *observable winning condition* is described by a pair (Ω, W_o), consisting of an observable colouring Ω and a set of infinite sequences of colours $W_o \subseteq C^\omega$. Then, the associated winning set is $W = \{v_0 v_1 v_2 \dots \mid \Omega(v_0)\Omega(v_1)\Omega(v_2)\dots \in W_o\}$.

A history is a finite prefix of a play. A *strategy* for Player i is a function $\sigma_i : V^* \to A_i$ such that $\sigma_i(\pi) = \sigma_i(\rho)$ for any two histories $\pi, \rho \in V^*$ with $\pi \sim_i^* \rho$, where $\sim_i^*$ is the extension of $\sim_i$ to sequences. A *joint strategy* for the grand coalition is a profile $\sigma = (\sigma_0, \dots, \sigma_{n-1})$ consisting of one strategy σ_i for every player i. We say that a play $\pi = v_0 v_1 \dots$ is *consistent* with σ, if $(v_l, \sigma(v_0 \dots v_l), v_{l+1}) \in \Delta$, for every stage $l > 0$. In this case, we refer to the histories of π as σ-histories. A joint strategy profile σ is *winning* from a position $v_0 \in V$, if each play from v_0 that is consistent with σ belongs to W. We study the following question: given a game $\mathcal{G}$, does the grand coalition have a winning strategy profile for $\mathcal{G}$?

▶ **Example 1.** Figure 1 describes a distributed game $\mathcal{G}_{\|}$ with two players. The relations $\sim_0$ and $\sim_1$ are represented by dashed and dotted lines, respectively. The game starts at position

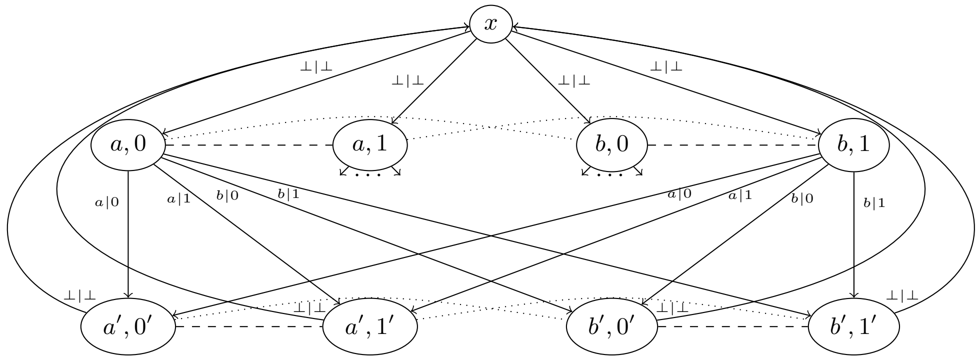


Figure 1 A distributed game $\mathcal{G}_{\|}$.

x where the players have only trivial moves $\bot$ and nature chooses a letter from $\{a, b\}$ and a digit from $\{0, 1\}$. The label of the successor position reflects this choice. Player 0 only observes whether nature has chosen a or b, whereas Player 1 observes whether it was 0 or 1. Next, Player 0 chooses a letter from $\{a, b\}$ and Player 1 a digit from $\{0, 1\}$, again reflected by the label of the successor. After that, the game returns to x for another round.

Let us set $\mathbb{A} = \{a, b\}$, $\mathbb{A}' = \{a', b'\}$, and $\mathbb{D} = \{0, 1\}$, $\mathbb{D}' = \{0', 1'\}$. As one player observes only letters and the other only digits, a strategy f of Player 0 in $\mathcal{G}$ corresponds to a function $(\mathbb{A}\mathbb{A}')^*\mathbb{A} \to \mathbb{A}$, whereas a strategy g of Player 1 corresponds to a function $(\mathbb{D}\mathbb{D}')^*\mathbb{D} \to \mathbb{D}$. Let W be a winning condition in $\mathcal{G}$, i.e. a subset of $(x(\mathbb{A} \times \mathbb{D})(\mathbb{A}' \times \mathbb{D}'))^*$. Then, the strategy profile (f, g) is winning if

$$x\binom{l_1}{d_1}\binom{f(l_1)'}{g(d_1)'}\, x\binom{l_2}{d_2}\binom{f(l_1\, f(l_1)'\, l_2)}{g(d_1\, g(d_1)'\, d_2)}\, x \cdots \in W.$$

Notice that, between two successive visits to position x, nature chooses a pair of bits and each of the players chooses one bit, with the first bit always revealed only to the first player and the second bit only to the second player. This is essentially the game structure considered in [13], where the authors construct regular winning conditions that require the players to construct the run of a given Turing machine. Deciding whether a winning profile (f, g) for the resulting game exists reduces to deciding whether the machine halts on the empty tape. Accordingly, the joint winning strategy problem is undecidable on this class of games.

2.2 Epistemic Models and Homomorphisms

To describe the knowledge acquired by the players during a play, we use epistemic models. An *epistemic model* over $\mathcal{G}$ is a Kripke structure $\mathcal{K} = (K, (P_v)_{v\in V}, (\sim_i)_{i<n})$ where $(P_v)_{v\in V}$ is a partition of K and each $\sim_i$ is an equivalence relation on K such that, for all $k, k' \in K$, if $k \sim_i k'$, then $v_k \sim_i v_{k'}$, with v_k denoting the unique element from V such that $k \in P_{v_k}$. Usually, $\mathcal{K}$ will be connected by $\sim_\cup = \bigcup_i \sim_i$, except when indicated otherwise. Notice that $\sim_\cup$ may not necessarily be an equivalence relation.

We recall the notion of graph homomorphism, which we apply to epistemic models. Let $\mathcal{K} = (K, (P_v)_{v\in V}, (\sim_i)_{i<n})$ and $\mathcal{K}' = (K', (P'_v)_{v\in V}, (\sim'_i)_{i<n})$ be epistemic models. A function f is a *homomorphism* from $\mathcal{K}$ to $\mathcal{K}'$, if $P_v(k) \implies P'_v(f(k))$ and $k \sim_i k' \implies f(k) \sim'_i f(k')$. The models are *homomorphically equivalent*, $\mathcal{K} \approx \mathcal{K}'$, if there exists a homomorphism from $\mathcal{K}$ to $\mathcal{K}'$ and one from $\mathcal{K}'$ to $\mathcal{K}$. Notice that $\approx$ is an equivalence relation as the composition of two homomorphisms is again a homomorphism.

For a finite epistemic model $\mathcal{K}$, a *core* is a model $\mathcal{K}' \approx \mathcal{K}$ with the minimal number of elements. One crucial observation, which follows for epistemic models in the same way as the standard argument for graphs, is that the core of a model is unique up to isomorphism.

▶ **Lemma 2.** *Every finite epistemic model has a unique core, up to isomorphism.*

3 Epistemic Unfolding

In more traditional approaches to analysing games on graphs, the unfolding collects histories of the original game. We present a new kind of unfolding that uses Kripke structures to collect the full description of the knowledge that players have at a certain stage of the play. When unfolding a game $\mathcal{G}$, we will keep track of the information available to all players in an epistemic model. Thus, the states of the unfolding are epistemic models over $\mathcal{G}$. At the start, we assume that all players know that they are at the initial position, thus the initial epistemic model will be a trivial, one-element structure consisting of $\{v_0\}$, $\mathcal{K}_0 = (\{v_0\}, (P_v)_{v\in V}, (\sim_i)_{i<n})$, where $P_{v_0} = \{v_0\}$, $P_w = \emptyset$ for $w \neq v_0$, and each $\sim_i = \{(v_0, v_0)\}$.

Assume that, in a state of the unfolding represented by an epistemic model $\mathcal{K}$, the players agreed on take actions described by a profile a. What will the epistemic state of a player be after executing these actions? Let $(a_k)_{k\in K}$ be a tuple of action profiles $a_k \in A$ compatible with the players' knowledge, i.e. for every $i < n$ and for all $k, k' \in K$ with $k \sim_i k'$, we have $(a_k)_i = (a_{k'})_i$. We define the, possibly disconnected, epistemic model

$$\mathrm{Update}(\mathcal{K}, (a_k)_{k\in K}) := (K', (P_v)_{v\in V}, (\sim_i)_{i<n}), \text{ by setting}$$

- $K' = \{kv \mid k \in K, k \in P_w \text{ and } (w, a_k, v) \in \Delta\}$,
- $P_v = \{kv \mid kv \in K'\}$,
- $kv \sim_i k'v' \iff k \sim_i^{\mathcal{K}} k'$ and $v \sim_i^{\mathcal{G}} v'$.

The set of epistemic successor models $\mathrm{Next}(\mathcal{K}, (a_k)_{k\in K})$ consists of the $\sim_\cup$-connected components of $\mathrm{Update}(\mathcal{K}, (a_k)_{k\in K})$.

To unfold a game $\mathcal{G}$ and track the knowledge with epistemic models, we start with the initial structure $\mathcal{K}_0$ as above and consider all possible action profiles a the players can take. We get the epistemic models $\mathrm{Next}(\mathcal{K}, a)$ as next states, and continue the unfolding from there. With this dynamic process in mind, we give the following declarative definition.

▶ **Definition 3** (Epistemic Unfolding)**.** The *epistemic unfolding* of a distributed game $\mathcal{G}$ is a game

$$\mathrm{Tr}(\mathcal{G}) := (V^t, \Delta^t, (\sim_i)_{i<n}, W^t), \text{ where}$$

- V^t is the set of all epistemic models $\mathcal{K}$ over $\mathcal{G}$ with $K \subseteq V^*$,
- $\Delta^t = \{(\mathcal{K}, (a_k)_{k\in K}, \mathcal{K}') \mid (a_k)_{k\in K} \in A^{|K|} \text{ and } \mathcal{K}' \in \mathrm{Next}(\mathcal{K}, (a_k)_{k\in K})\}$,
- $\sim_i = \{(\mathcal{K}, \mathcal{K}) \mid \mathcal{K} \in V^t\}$, i.e. $\mathrm{Tr}(\mathcal{G})$ is a game with perfect information,
- $\mathcal{K}_0\mathcal{K}_1 \cdots \in W^t$ if, and only if, for *each* sequence $\pi = k_0k_1\ldots$ such that $k_l \in \mathcal{K}_l$ and $k_{l+1} = k_lv$ for some v, with $(v_{k_l}, a, v) \in \Delta$ for some a, it holds that $v_{k_0}v_{k_1}\cdots \in W$.

Note that the actions in the game $\mathrm{Tr}(\mathcal{G})$ correspond to tuples of actions in the original game: At a position $\mathcal{K}$, each player i chooses a tuple of actions $((a_k)_i)_{k\in K}$, one for every world of the epistemic model $\mathcal{K}$. The tuple of action profiles $(a_k)_{k\in K}$ yields the set $\mathrm{Next}(\mathcal{K}, (a_k)_{k\in K})$ of successor models from which nature chooses the next position.

The winning condition of $\mathrm{Tr}(\mathcal{G})$ requires that all paths through the sequence of Kripke structures be winning in the original game. Let us detail this for the case of observable

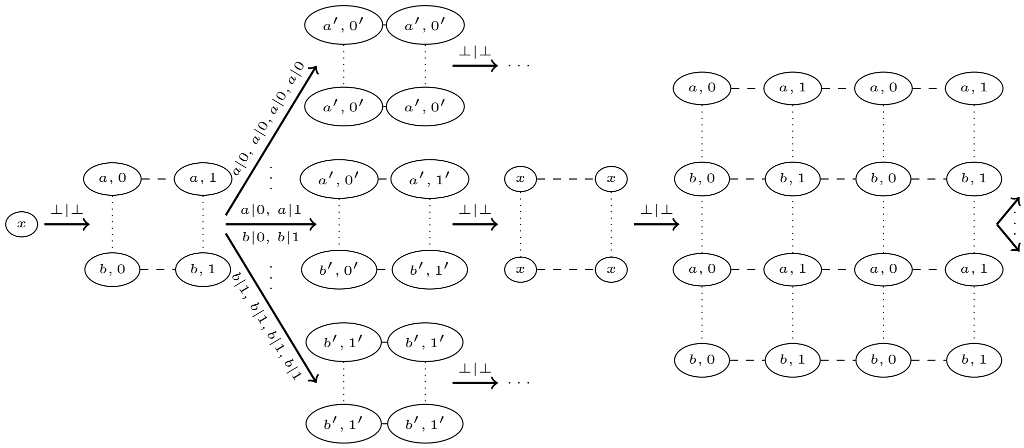


Figure 2 Epistemic unfolding $\mathrm{Tr}(\mathcal{G}_{\|})$ of the game $\mathcal{G}_{\|}$.

winning conditions (Ω, W_o). Since epistemic models are $\sim_{\cup}$-connected, the colouring Ω is constant for all worlds of a position $\mathcal{K} \in \mathrm{Tr}(\mathcal{G})$; we write $\Omega(\mathcal{K})$ for this colour. Then, we have $\mathcal{K}_0\mathcal{K}_1 \cdots \in W^t$ if, and only if, $\Omega(\mathcal{K}_1)\Omega(\mathcal{K}_2)\ldots \in W_o$. Notice however, that this description is not valid for infinite game graphs, as there may be infinite plays in $\mathrm{Tr}(\mathcal{G})$ for which there is no corresponding infinite play in $\mathcal{G}$. In the case of finite game graphs the above remark follows by König's Lemma.

Observe that, since $\mathrm{Tr}(\mathcal{G})$ is a game with perfect information, in particular all players of the grand coalition have the same information. Thus, the grand coalition can be regarded as a single super-player who chooses actions on behalf of every member of the coalition, and the game can be solved as if it was a two-player game between this super-player and nature (now regarded as a second player).

▶ **Example 4.** In Figure 2, we represent a few first steps of the epistemic unfolding $\mathrm{Tr}(\mathcal{G}_{\|})$ of the game $\mathcal{G}_{\|}$ from Example 1. Note that the structures get larger as more and more knowledge of the players has to be accounted for. Also observe that, in contrast to the standard unfolding, the branching factor in $\mathrm{Tr}(\mathcal{G}_{\|})$ may grow with increasing level.

The following theorem explains the basic utility of the epistemic unfolding.

▶ **Theorem 5.** *The grand coalition has a winning strategy in the distributed game $\mathcal{G}$ from v_0 if, and only if, the grand coalition has a winning strategy in $\mathrm{Tr}(\mathcal{G})$ from $\mathcal{K}_0$.*

Proof. ($\Rightarrow$) First, let $\sigma = (\sigma_0, \ldots, \sigma_{n-1})$ be a winning strategy for the coalition in $\mathcal{G}$ from v_0. We define the strategy $\sigma^t = (\sigma_0^t, \ldots, \sigma_{n-1}^t)$ for the coalition for $\mathrm{Tr}(\mathcal{G})$ by induction over the length of histories of $\mathrm{Tr}(\mathcal{G})$ from $\mathcal{K}_0$ such that, for each history $\pi = \mathcal{K}_0 \ldots \mathcal{K}_r$ consistent with σ^t, every $\pi \in K_r$ is consistent with σ. Note that, in each step r, we only need to extend σ^t to histories of length $r+1$ that are consistent with σ^t. For $r = 0$ the statement is trivial. Let now $\pi^t = \mathcal{K}_0 \ldots \mathcal{K}_r$ be an arbitrary history of $\mathrm{Tr}(\mathcal{G})$ that is consistent with σ^t. We define $\sigma^t(\pi^t) = (a_k)_{k \in K_r}$ by setting $a_k = \sigma(k)$, for every $k \in K$. Notice that each $k \in K_r$ is a σ-history of $\mathcal{G}$ from v_0. We observe that $(a_k)_{k \in K_r} \in \mathrm{act}(\mathcal{K}_r)$: If $k \sim_i k'$, then $k \sim_i^* k'$, and since σ_i is a strategy for player i for $\mathcal{G}$, we have $(a_k)_i = \sigma_i(k) = \sigma_i(k') = (a_{k'})_i$. Now consider a model $\mathcal{K}_{r+1} \in \mathrm{Next}(\mathcal{K}_r, (a_k)_{k \in K_r})$. By definition, $\pi^t\mathcal{K}_{r+1}$ is consistent with σ^t and every $\pi \in K_{r+1}$ is consistent with σ. This concludes the induction argument.

Next, consider any play $\pi^t = \mathcal{K}_0\mathcal{K}_1\dots$ in $\mathrm{Tr}(\mathcal{G})$ from $\mathcal{K}_0$ consistent with σ^t. Let now $\rho = k_0k_1\dots$ be any path through the structures in π^t. Since $k_0 = v_0$ and, by construction, each $k_i = v_0\dots v_i$ such that $v_0\dots v_i$ is a history consistent with σ, we get that $v_0v_1\dots \in W$, and thus $\pi^t \in W^t$, by definition. Hence, σ^t is a winning strategy.

($\Leftarrow$) Now let $\sigma^t = (\sigma^t_0, \dots, \sigma^t_{n-1})$ be a winning strategy for the coalition in $\mathrm{Tr}(\mathcal{G})$ from $\mathcal{K}_0$. We define the strategy $\sigma = (\sigma_0, \dots, \sigma_{n-1})$ for the coalition for $\mathcal{G}$ by induction over the length of histories of $\mathcal{G}$ from $\mathcal{K}_0$ and, simultaneously, with each σ-history $\pi = v_0\dots v_r$ of $\mathcal{G}$, we associate a history $\zeta(\pi) = \mathcal{K}_0\dots\mathcal{K}_r$ of $\mathrm{Tr}(\mathcal{G})$ from v_0, such that the following holds.

(i) $\pi \in K_r$;
(ii) if $\rho \sim_i^* \pi$ for some σ-history ρ in $\mathcal{G}$ from v_0 and some $i < n$, then $\zeta(\rho) = \zeta(\pi)$;
(iii) $\zeta(\pi)$ is consistent with σ^t;
(iv) $\zeta(v_0\dots v_l) = \mathcal{K}_0\dots\mathcal{K}_l$ for any $l \le r$.

Note that in each step r, we only need to extend σ to histories of length $r+1$ that are consistent with σ. For $\pi = v_0$ we take $\mathcal{K}_0$ as defined before. Now let $\pi = v_0\dots v_r$ be any history of $\mathcal{G}$ from v_0 that is consistent with σ and let $\zeta(\pi) = \mathcal{K}_0\dots\mathcal{K}_r$. We define $\sigma_i(\pi) = (a_\pi)_i$, where $(a_k)_{k\in K_r} := \sigma^t(\zeta(\pi))$, that means, $\sigma_i(\pi)$ is the projection to the i-th component of the action, chosen by player i at $\zeta(\pi)$ for the position $\pi \in K_r$ according to σ^t. First, we observe that σ_i is constant over $\sim_i^*$-equivalence classes: if $\rho \sim_i^* \pi$ for some σ-history ρ of $\mathcal{G}$ from v_0, then by condition (i) and (ii) we have $\rho \in \zeta(\rho) = \zeta(\pi)$, so $\sigma_i(\rho) = (a_\rho)_i$. Moreover, as $\pi \sim_i^* \rho$ and $(a_k)_{k\in K_r} \in \mathrm{act}(\mathcal{K}_r)$, $(a_\pi)_i = (a_\rho)_i$.

Now let $v_{r+1} \in V$ such that $(v_r, \sigma(\pi), v_{r+1}) \in \Delta$ (i.e., πv_{r+1} is a σ-history) and let $\mathcal{K}_{r+1} \in \mathrm{Next}(\mathcal{K}_r, (a_k)_{k\in K_r})$ such that $\pi v_{r+1} \in K_{r+1}$, that means, $\mathcal{K}_{r+1}$ is the unique $\sim_\cup$-connected component of the epistemic model $\mathrm{Update}(\mathcal{K}, (a_k)_{k\in K_r})$ that contains πv_{r+1}. Observe that, since $\mathrm{last}(\pi) = v_r$ and $(v_r, a_\sigma, v_{r+1}) \in \Delta$, the history πv_{r+1} is contained in $\mathrm{Update}(\mathcal{K}, (a_k)_{k\in K_r})$, ensuring (i) and by induction (iv). By definition, $\zeta(\pi v_{r+1}) = \zeta(\pi)\mathcal{K}_{r+1}$ is consistent with σ^t, ensuring (iii), so it remains to show (ii), i.e. that if $\rho v \sim_i^* \pi v_{r+1}$ for some σ-history ρv of $\mathcal{G}$ from v_0 and some $i < n$, then $\zeta(\rho v) = \zeta(\pi v_{r+1})$.

First, notice that $\rho v \sim_i^* \pi v_{r+1}$ implies $\rho \sim_i^* \pi$, so $\zeta(\rho) = \zeta(\pi)$. Moreover, the construction of $\zeta(\pi v_{r+1})$ from $\zeta(\pi) = \zeta(\rho)$ is independent of πv_{r+1}, except for the choice of the $\sim_\cup$-connected component $\mathcal{K}_{r+1} \in \mathrm{Next}(\mathcal{K}_r, a)$ of $\mathrm{Update}(\mathcal{K}, (a_k)_{k\in K_r})$. As ρv is a σ-history with $\rho \in \mathcal{K}_r$, by definition of $\sigma(\rho)$, we have $\rho v \in \mathrm{Update}(\mathcal{K}_r, (a_k)_{k\in K_r})$, since $\rho v \sim_i^* \pi v_{r+1}$, ρv and πv_{r+1} lie in the same $\sim_\cup$-connected component of $\mathrm{Update}(\mathcal{K}_r, (a_k)_{k\in K_r},)$.

Finally, consider any play $\pi = v_0v_1\dots$ in $\mathcal{G}$ from v_0 that is consistent with σ and let $\pi^t = \mathcal{K}_0\mathcal{K}_1\dots$ be the play in $\mathrm{Tr}(\mathcal{G})$ from $\mathcal{K}_0$ associated with π, i.e. $\zeta(v_0\dots v_l) = \mathcal{K}_0\dots\mathcal{K}_l$ for all l. By construction, any finite prefix $v_0v_1\dots v_l$ is also a path through π^t of the form $k_0k_1\dots k_l$, and this extends to the whole play π. Since π^t is consistent with σ^t and thus won by the coalition, by definition of the winning condition W^t, we get that $\pi \in W$. ◀

4 Epistemic Unfolding up to Homomorphic Equivalence

We turn to the task of representing the game $\mathrm{Tr}(\mathcal{G})$ more succinctly. One simple approach would be to identify isomorphic epistemic models; then, strategies can be transferred by isomorphism. To obtain a more significant degree of succinctness, we show that, if the winning condition is observable, it is sufficient to distinguish epistemic models up to homomorphic equivalence. Consequently, we may take the core of each model (or any retract) instead of the model itself while unfolding.

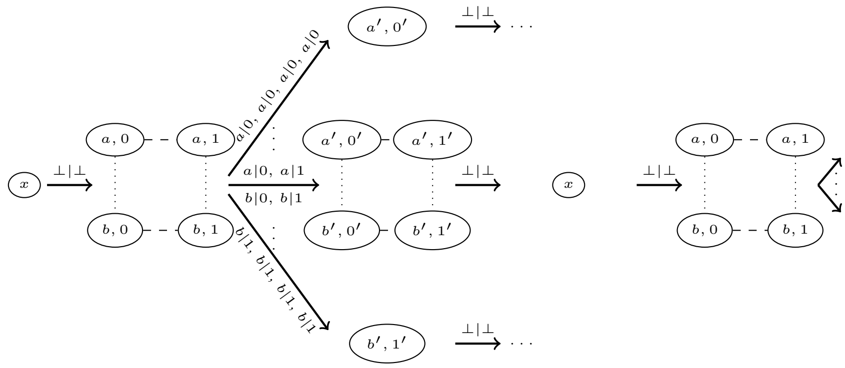


Figure 3 Epistemic unfolding $\mathrm{Tr}(\mathcal{G}_{\|})$ quotiented by core.

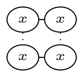
Essentially, epistemic unfolding up to homomorphism consists of performing the tracking construction while identifying homomorphically equivalent models. Since there may be many possible models equivalent to a model $\mathcal{K}$, we describe this unfolding with respect to a function q, defined on all epistemic models, which chooses for every model $\mathcal{K}$ a homomorphically equivalent companion model $q(\mathcal{K}) \approx \mathcal{K}$.

Although unfolding up to homomorphism is sound only for observable winning conditions (Ω, W_o), we first define the notion for arbitrary winning conditions W. As in the case of the tracking $\mathrm{Tr}(\mathcal{G})$ for games with observable winning conditions, the following definition can be phrased equivalently using sets of colour sequences $W_o \in C^\omega$ to describe winning conditions.

▶ **Definition 6** (Epistemic Unfolding up to Homomorphic Equivalence).
The epistemic unfolding of a distributed $\mathcal{G}$ *up to homomorphic equivalence*, with respect to a function q, is a game

$$\mathrm{Tr}^q(\mathcal{G}) := (V^q, \Delta^q, (\sim_i)_{i<n}, W^q), \text{ where}$$

- V^q is the set $\{q(\mathcal{K}) \mid \mathcal{K}$ is an epistemic model over $\mathcal{G}\}$,
- $\Delta^q = \{(\mathcal{K}, (a_k)_{k\in K}, q(\mathcal{K}')) \mid (a_k)_{k\in K} \in A^{|K|}$ and $\mathcal{K}' \in \mathrm{Next}(\mathcal{K}, (a_k)_{k\in K})\}$,
- $\sim_i = \{(\mathcal{K}, \mathcal{K}) \mid \mathcal{K} \in V^q\}$, i.e. $\mathrm{Tr}^q(\mathcal{G})$ is a game with perfect information,
- $\mathcal{K}_0\mathcal{K}_1 \cdots \in W^q$ if, and only if, for *each* sequence $\pi = k_0k_1\ldots$ such that $k_l \in \mathcal{K}_l$ and $k_{l+1} = q(k_l v)$ for some v, with $(v_{k_l}, a, v) \in \Delta$ for some a, it holds that $v_{k_0}v_{k_1}\cdots \in W$.

▶ **Example 7.** We are particularly interested in the case when the image of the homomorphism is the core, i.e., $q(\mathcal{K}) = \mathrm{core}(\mathcal{G})$. In Figure 2, we presented a few positions from the epistemic unfolding $\mathrm{Tr}(\mathcal{G}_{\|})$. In Figure 3 we present the same situation, but these structures are now replaced by their cores. Note that, for example,  gets quotiented to  and thus, from the fourth stage, the structures are repeated. Since we identify isomorphic Kripke structures, the game $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G}_{\|})$ is a finite game with perfect information.

Note that, since $\mathcal{K} \approx \mathcal{K}$, the unfolding Tr^q is a generalisation of the tracking construction Tr obtained with $q(\mathcal{K}) = \mathcal{K}$. We will extend Theorem 5 to all unfoldings Tr^q for games with observable winning conditions. The key point is how to extend the homomorphisms from a model to the next one in a tracking. This is an interesting observation in itself, we formulate it as a separate lemma.

▶ **Lemma 8.** *Let $\mathcal{K}$ and $\mathcal{L}$ be epistemic models, let $h : \mathcal{K} \to \mathcal{L}$ be a homomorphism, and let $(b_l)_{l \in L}$ be a tuple of actions for $\mathcal{L}$. Then $(a_k)_{k \in K}$ with $a_k = b_{h(k)}$ is a tuple of actions for $\mathcal{K}$, and for each connected component $\mathcal{K}'$ of $\mathrm{Update}(\mathcal{K}, (a_k)_{k \in K})$, there is a connected component $\mathcal{L}'$ of $\mathrm{Update}(\mathcal{L}, (b_l)_{l \in L})$ such that there is a homomorphism $h' : \mathcal{K}' \to \mathcal{L}'$.*

Proof. Since h is a homomorphism, $(a_k)_{k \in K}$ is obviously a tuple of actions for $\mathcal{K}$. Let $\mathcal{K}'$ be a connected component of $\mathrm{Update}(\mathcal{K}, (a_k)_{k \in K})$ and consider the connected component of $\mathrm{Update}(\mathcal{L}, (b_l)_{l \in L})$ that contains all elements $h(k)v$ with $kv \in K'$. Note that since $\mathcal{K}'$ is connected by $\sim_\cup$ and h is a homomorphism, the elements $h(k)v$ are $\sim_\cup$-connected as well and thus are included in a single $\mathcal{L}'$, which we denote by $h(\mathcal{K}')$. The mapping $h' : \mathcal{K}' \to \mathcal{L}'$ with $h'(kv) = h(k)v$ is again a homomorphism, now from $\mathcal{K}'$ to $\mathcal{L}'$. ◀

▶ **Theorem 9.** *Let $\mathcal{G}$ be a distributed game with observable winning condition (Ω, W_o). Then, for all q, the following are equivalent.*

(1) The grand coalition has a winning strategy for $\mathcal{G}$ from v_0.
(2) The grand coalition has a winning strategy for $\mathrm{Tr}(\mathcal{G})$ from $\mathcal{K}_0$.
(3) The grand coalition has a winning strategy for $\mathrm{Tr}^q(\mathcal{G})$ from $q(\mathcal{K}_0)$.

Proof. The equivalence of (1) and (2) was shown already in Theorem 5.

(2) $\Rightarrow$ (3) Let σ^t be a joint winning strategy for the grand coalition in $\mathrm{Tr}(\mathcal{G})$. We define the joint winning strategy σ^q for the grand coalition in $\mathrm{Tr}^q(\mathcal{G})$ by induction on the length of histories in $\mathrm{Tr}^q(\mathcal{G})$ and simultaneously, with each such history $\pi^q = \mathcal{L}_0\mathcal{L}_1 \dots \mathcal{L}_r$ that is consistent with σ^q, we associate a history $\mu(\pi^q) = \mathcal{K}_0\mathcal{K}_1 \dots \mathcal{K}_r$ in $\mathrm{Tr}(\mathcal{G})$, such that the following conditions hold:

(i) $\mu(\pi^q)$ is consistent with σ^t;
(ii) there is a homomorphism $\nu : \mathcal{L}_r \to \mathcal{K}_r$;
(iii) $\mu(\mathcal{L}_0\mathcal{L}_1 \dots \mathcal{L}_s) = \mathcal{K}_0\mathcal{K}_1 \dots \mathcal{K}_s$ for each $s \le r$.

For $r = 1$, there is only one history $\pi^q = \mathcal{L}_0 = q(\mathcal{K}_0)$, thus $\mu(\pi^q) = \mathcal{K}_0$ and the homomorphism $\nu : \mathcal{L}_0 \to \mathcal{K}_0$ is obtained from $\mathcal{K}_0 \approx q(\mathcal{K}_0)$. In the following, for an epistemic model $\mathcal{K}$, let $\varphi_\mathcal{K}$ always denote a homomorphism $\varphi : q(\mathcal{K}) \to \mathcal{K}$; in this notation we write $\nu = \varphi_{\mathcal{K}_0}$. Let now $\pi^q_r = \mathcal{L}_0 \dots \mathcal{L}_r$ be a history consistent with σ^q, let $\mu(\pi^q_r) = \mathcal{K}_0 \dots \mathcal{K}_r$ be the associated history consistent with σ^t, and let $\nu : \mathcal{L}_r \to \mathcal{K}_r$ be a homomorphism according to (ii). Consider the actions $(a_k)_{k \in K_r} = \sigma^t(\mu(\pi^q_r))$ prescribed by σ^t, given the history $\mu(\pi^q_r)$ in the game $\mathrm{Tr}(\mathcal{G})$. We define $\sigma^q(\pi^q_r)$ by $\sigma^q(\pi^q_r)(l) = a_{\nu(l)} = \sigma^t(\mu(\pi^q_r))(\nu(l))$. By Lemma 8, $\sigma^q(\pi^q_r)$ is a tuple of actions for $\mathcal{L}_r$. So, for any connected component $\mathcal{L}'$ of $\mathrm{Update}(\mathcal{L}_r, \sigma^q(\pi^q_r))$, the sequence $\pi^q_{r+1} = \pi^q_r \mathcal{L}_{r+1}$ with $\mathcal{L}_{r+1} = q(\mathcal{L}')$ is a history of $\mathrm{Tr}^q(\mathcal{G})$ that is, by definition, consistent with σ^q. Moreover, Lemma 8 yields a homomorphism $\eta : \mathcal{L}' \to \mathcal{K}'$ for a connected component $\mathcal{K}'$ of $\mathrm{Update}(\mathcal{K}_r, \sigma^t(\mu(\pi^q)))$. So, by composing the homomorphism $\varphi_{\mathcal{L}_{r+1}}$ from $\mathcal{L}_{r+1}$ to $q(\mathcal{L}_{r+1}) = \mathcal{L}'$ with the homomorphism η from $\mathcal{L}'$ to $\mathcal{K}'$ we obtain a homomorphism $\nu' : \mathcal{L}_{r+1} \to \mathcal{K}'$ and we set $\mu(\pi^q_{r+1}) = \mu(\pi^q_r)\mathcal{K}'$. By construction, $\mu(\pi^q_{r+1})$ is consistent with σ^t.

Now let $\pi^q = \mathcal{L}_0\mathcal{L}_1 \dots$ be a play in $\mathrm{Tr}^q(\mathcal{G})$ consistent with σ^q. By (iii), the sequence $\mu(\mathcal{L}_0), \mu(\mathcal{L}_0\mathcal{L}_1), \dots$ yields a play $\mathcal{K}_0\mathcal{K}_1 \dots$ in $\mathrm{Tr}(\mathcal{G})$ consistent with σ^t such that, for each $r \in \mathbb{N}$, there is a homomorphism $\nu_r : \mathcal{L}_r \to \mathcal{K}_r$. To show that π^q is indeed a winning play, consider any sequence $\pi = l_0 l_1 \dots$ with $l_r \in L_r$ and $l_{r+1} = q(l_r v)$ for v with $(v_{l_r}, a, v) \in \Delta$ for some a. In particular, $\nu_r(l_r) \in K_r$ for each $r \in \mathbb{N}$, so $\Omega(v_{l_0})\Omega(v_{l_1}) \dots = \Omega(\mathcal{K}_0)\Omega(\mathcal{K}_1) \dots$ and as $\mathcal{K}_0\mathcal{K}_1 \dots \in W^q$ we have $\Omega(v_{l_0})\Omega(v_{l_1}) \dots \in W_o$, which proves that π^q is winning. Hence, σ^q is a winning strategy for the grand coalition.

(3) ⇒ (2) This direction follows analogously, by symmetry of homomorphic equivalence. ◀

Notice that, despite the symmetric argument in the proof, if q maps an epistemic model to its core, then $\varphi : \mathcal{K} \to q(\mathcal{K})$ is surjective while $\varphi : q(\mathcal{K}) \to \mathcal{K}$ is injective. This allows to prove the implication from (3) to (2) even in the case of winning conditions that are not observable. However, the implication from (2) to (3) does not hold in general.

While the theorem above can be applied to an arbitrary tracking Tr^q such that $q(\mathcal{K}) \approx \mathcal{K}$, we will concentrate on a specific one, namely $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$, obtained with the function that maps every structure $\mathcal{K}$ to its core. The uniqueness of a core allows us to prove the following remarkable property.

▶ **Theorem 10.** *There exists a finite tracking $\mathrm{Tr}^q(\mathcal{G})$ of $\mathcal{G}$ if, and only if, the tracking $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$ is finite.*

As a consequence, we obtain a semi-decision procedure for solving distributed games with observable winning conditions: compute $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$ and if it is finite, solve the resulting game with perfect information. The procedure thus takes arbitrary games with observable winning condition as input. This is in contrast with tree-automata based methods, which require a certain information-order among the players, and hence a-priori restrict possible inputs.

5 Hierarchical Games

We present an application of our construction to hierarchical games, which were studied in [18] and are related to the ones in [13, 7]. In particular, they subsume observable ω-regular games with imperfect information where one player has perfect information.

A game $\mathcal{G} = (V, \Delta, (\sim_i)_{i<n}, W)$ is *hierarchical*, if $\sim_0 \subseteq \sim_1 \subseteq \ldots \subseteq \sim_{n-1}$, i.e., if the knowledge of the players is ordered linearly: Player 0 is the best informed one, and Player $n-1$ knows the least. The following theorem provides us with a bound on the size of the game of perfect information obtained by the epistemic unfolding up to homomorphic equivalence of a hierarchical game with imperfect information.

▶ **Theorem 11.** *Let V be a finite set and $n \in \mathbb{N}$. Up to homomorphic equivalence, there are at most $\exp_n(|V|)$ different Kripke structures $\mathcal{K} = (K, (P_v)_{v\in V}, (\sim_i)_{i<n})$ such that:*

1. *$(P_v)_{v\in V}$ is a partition of K*
2. *$\sim_1 \subseteq \ldots \subseteq \sim_n$ are equivalence relations*
3. *$\mathcal{K}$ is connected by $\bigcup_{i=1}^{n} \sim_i$.*

Proof. We denote by $\Psi_n(V)$ the class of all Kripke structures $\mathcal{K} = (K, (P_v)_{v\in V}, (\sim_i)_{i<n})$ with the properties 1. - 3, and we write $\sim_{\cup}^{n} := \bigcup_{i=0}^{n-1} \sim_i$. We prove by induction that, for each $n \in \mathbb{N}$, there is a class $\Psi_n^{\approx}(V)$ of Kripke structures from $\Psi_n(V)$ with $|\Psi_n^{\approx}(V)| = \exp_n(|V|)$ such that each structure from $\Psi_n(V)$ is homomorphically equivalent to one from $\Psi_n^{\approx}(V)$.

First, we define $\Psi_1^{\approx}(V)$ as the set of all Kripke structures $\mathcal{K} = (K, (P_v)_{v\in V}, \sim_0)$ with $K \subseteq V$, $P_v = \{v\}$ for $v \in V$ and $\sim_0 = K \times K$. Hence, any structure in $\Psi_1^{\approx}(V)$ can be identified with a subset of V, $|\Psi_1^{\approx}(V)| = 2^{|V|} = \exp_1(|V|)$. Clearly, $\Psi_1^{\approx}(V) \subseteq \Psi_1(V)$. Let $\mathcal{L} = (L, (P_v)_{v\in V}, \sim_0)$ be any Kripke structure from $\Psi_1(V)$. This structure is connected by $\sim_0 = L \times L$ and we define a homomorphism ν on $\mathcal{L}$ by $\nu(l) = v$, for the unique $v \in V$ such that $l \in P_v$. The homomorphic image $\nu(\mathcal{L}) = (K, (P_v)_{v\in V}, \sim_0)$ of ν is in $\Psi_1^{\approx}(V)$ and $\eta : \nu(\mathcal{L}) \to \mathcal{L}$ with $\eta(v) = l$ for some $l \in L \cap P_v$ is a homomorphism on $\nu(\mathcal{L})$. Hence, $\mathcal{L}$ and $\nu(\mathcal{L})$ are homomorphically equivalent.

For $n > 1$, suppose $\Psi^{\approx}_{n-1}(V)$ has already been constructed. Without loss, we assume that all Kripke structures from $\Psi^{\approx}_{n-1}(V)$ are pairwise disjoint. We define $\Psi^{\approx}_{n}(V)$ as the set of all Kripke structures $\mathcal{K} = (K, (P_v)_{v\in V}, (\sim_i)_{i<n})$ that consist of a union of epistemic models from $\Psi^{\approx}_{n-1}(V)$ and we set $\sim_{n-1} = K \times K$. Hence, any structure in $\Psi^{\approx}_{n}(V)$ can be identified with a subset of $\Psi^{\approx}_{n-1}(V)$, so $|\Psi^{\approx}_{n}(V)| = 2^{\exp_{n-1}(|V|)} = \exp_n(|V|)$. Now, let $\mathcal{L} = (L, (P_v)_{v\in V}, (\sim_i)_{i<n})$ be any Kripke structure from $\Psi_n(V)$. As $\mathcal{L}$ is connected by $\sim^n_{\cup}$, we have $\sim_{n-1} = L \times L$: any $l, l' \in L$ are connected in $\mathcal{L}$ via some $\sim^n_{\cup}$-path and as $\sim^{n-1}_{\cup} \subseteq \sim_{n-1}$ and $\sim_{n-1}$ is transitive, it follows that $l \sim_{n-1} l'$.

Consider the decomposition of $\mathcal{L}$ into $\sim^{n-1}_{\cup}$-connected components $\mathcal{L}_1, \dots, \mathcal{L}_r$. Clearly, $\mathcal{L}_j \in \Psi_{n-1}(V)$ for $j = 1, \dots, r$ and hence, each $\mathcal{L}_j$ is homomorphically equivalent to a Kripke structure from $\Psi^{\approx}_{n-1}(V)$. For $j \in \{1, \dots, r\}$ we fix a homomorphism ν_j on $\mathcal{L}_j$ such that the image $\nu_j(\mathcal{L}_j)$ is in $\Psi^{\approx}_{n-1}(V)$ and a homomorphism η_j from $\nu_j(\mathcal{L}_j)$ to $\mathcal{L}_j$. Moreover, we define the homomorphism ν on $\mathcal{L}$ by $\nu{\restriction}\mathcal{L}_j = \nu_j$ for $j = 1, \dots, r$. As the components $\mathcal{L}_j$ are pairwise disjoint, this is well defined and it is easy to see that the homomorphic image $\nu(\mathcal{L})$ is in $\Psi^{\approx}_{n}(V)$. Furthermore, we define $\eta : \nu(\mathcal{L}) \to \mathcal{L}$ as follows. For any $\sim^{n-1}_{\cup}$-connected component $\mathcal{M}$ of $\nu(\mathcal{L})$ there exists $j \in \{1, \dots, r\}$ such that $\nu_j(\mathcal{L}_j) = \mathcal{M}$ and we define $\eta{\restriction}\mathcal{M} = \eta_j$, for arbitrary j. Now, η is a homomorphism and hence, $\mathcal{L}$ and $\nu(\mathcal{L})$ are homomorphically equivalent. ◀

▶ **Corollary 12.** *For hierarchical games with observable regular winning conditions, the existence of a joint winning strategy for the grand coalition is decidable.*

Proof. Let $\mathcal{G}$ be a hierarchical game with an observable winning condition (Ω, W_o) such that W_o is regular. By Theorem 9, the grand coalition has a joint winning strategy for $\mathcal{G}$ if, and only if, it has a joint winning strategy for $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$, and by Theorem 11, $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$ is finite. Moreover, as we observed previously, the winning condition of $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$ can be described as $W^{\mathrm{core}} = \{\mathcal{L}_0\mathcal{L}_1 \ldots \in (V^{\mathrm{core}})^\omega \mid \Omega(\mathcal{L}_0)\Omega(\mathcal{L}_1)\ldots \in W_o\}$ so W^{core} is regular as well. Hence, the existence of a joint winning strategy in $\mathcal{G}$ can be decided by solving the game $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$ – a finite game with perfect information and a regular winning condition. ◀

6 Outlook

We introduced the epistemic unfolding $\mathrm{Tr}(\mathcal{G})$ of a distributed game $\mathcal{G}$ to capture the knowledge of players; the resulting structure is infinite for all games $\mathcal{G}$ of infinite duration. To obtain a more succinct representation, we restrict to the core of the generated epistemic models and obtain perfect information games $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$ that are finite for certain game instances $\mathcal{G}$, in particular for all hierarchical ones. However, we can only guarantee that the quotient $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$ preserves winning strategies if the winning condition of $\mathcal{G}$ is observable. Nevertheless, even under observable winning conditions there exist distributed games for which it is undecidable whether a winning strategy profile exists (cf. Propositions 22-24 in [2]).

Theorem 9 and Lemma 8 demonstrate that homomorphic equivalence allows to transfer strategies in observable games. We are persuaded that homomorphic equivalence – and *not* bisimulation – is a suitable notion for a quotient. The current work brought us to the insight that the bisimulation-based tracking introduced in our previous paper [2] does not preserve winning strategies – the assertion of Lemma 14 of the paper is incorrect in the stated form. We are currently preparing an erratum communication on this result, where we will also discuss the appropriateness of homomorphic equivalence in more detail.

The construction of $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$ can be done on the fly. Thus, our result provides a semi-decision procedure for the coordinated winning strategy problem for games with imperfect information and observable winning conditions. The procedure halts on all hierarchical games. One important task is to characterise further game classes that are solvable in this way, i.e. instances $\mathcal{G}$ for which $\mathrm{Tr}^{\mathrm{core}}(\mathcal{G})$ is finite. Another challenge is to develop a similar semi-decision procedure for games with non-observable winning conditions. Does there exists a uniform quotienting function q such that $\mathrm{Tr}^{q}(\mathcal{G})$ preserves winning strategies for all (even non-observable) winning conditions and is finite for hierarchical games? If not, does such a quotient q_W exists for each regular (non-observable) winning condition W separately?

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