Algorithmic Aspects of Semiring Provenance for Stratified Datalog

Matthias Naaf

Logic and Algebra for Query Evaluation, Berkeley 2023

Algorithmic Aspects

Semiring Provenance for Stratified Datalog

Algorithmic Aspects

Semiring Provenance for Stratified Datalog

Greatest Fixed Points

(in absorptive semirings)

Algorithmic Aspects

Semiring Provenance for Stratified Datalog

Computing Greatest Fixed Points

(in absorptive semirings)

Algorithmic Aspects

Semiring Provenance for Stratified Datalog

Computing Greatest Fixed Points

(in absorptive semirings)

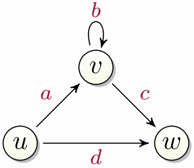
Circuit Representations

Why Greatest Fixed Points?

Semiring Semantics for Datalog

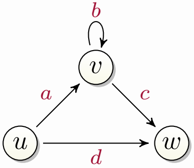
Datalog

$$Txy \ \text{:–}\ Exy$$
$$Txy \ \text{:–}\ Exz, Tzy$$



Datalog

$$Txy \;{:}{-}\; Exy$$
$$Txy \;{:}{-}\; Exz, Tzy$$



$\Rightarrow$

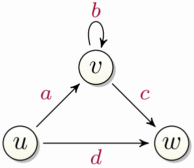
Equation System

$$T_{uv} = a \vee (a \wedge T_{vv}) \vee (d \wedge T_{wv})$$
$$T_{uw} = d \vee (d \wedge T_{ww}) \vee (a \wedge T_{vw})$$
$$\vdots$$

Semiring Semantics for Datalog

Datalog

$$Txy \;{:}\!-\; Exy$$
$$Txy \;{:}\!-\; Exz, Tzy$$



$\Rightarrow$

Equation System

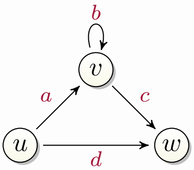
$$T_{uv} = a + (a \cdot T_{vv}) + (d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww}) + (a \cdot T_{vw})$$
$$\vdots$$

Semiring Semantics for Datalog

Datalog

$Txy \;{:}\!-\; Exy$

$Txy \;{:}\!-\; Exz, Tzy$



$\Rightarrow$

Equation System

$$T_{uv} = a + (a \cdot T_{vv}) + (d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww}) + (a \cdot T_{vw})$$
$$\vdots$$

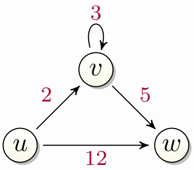
Semantics: Least solution

- Power series: $T^*_{uw} = d + ac + abc + ab^2c + ab^3c + \dots$
- PosBool: $T^*_{uw} = d \vee (a \wedge c)$

Semiring Semantics for Datalog

Datalog

$$Txy \coloneq Exy$$
$$Txy \coloneq Exz, Tzy$$



$\Rightarrow$

Equation System

$$T_{uv} = a + (a \cdot T_{vv}) + (d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww}) + (a \cdot T_{vw})$$
$$\vdots$$

Semantics: Least solution

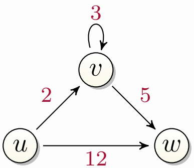
- Power series: $T^*_{uw} = d + ac + abc + ab^2c + ab^3c + \dots$
- PosBool: $T^*_{uw} = d \vee (a \wedge c)$
- Tropical: $T^*_{uw} = \min(12, 2 + 5) = 7$

(Green, Karvounarakis, Tannen, PODS'07)

Semiring Semantics for Datalog

Datalog

$$Txy \;\text{:–}\; Exy$$
$$Txy \;\text{:–}\; Exz, Tzy$$



$\Rightarrow$

Equation System

$$T_{uv} = a + (a \cdot T_{vv}) + (d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww}) + (a \cdot T_{vw})$$
$$\vdots$$

Semantics: Least solution

- Power series: $T^*_{uw} = d + ac + abc + ab^2c + ab^3c + \ldots$
- PosBool: $T^*_{uw} = d \vee (a \wedge c)$
- Tropical: $T^*_{uw} = \min(12, 2 + 5) = 7$

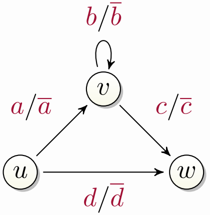
ω-continuous semirings

Semiring Semantics for Stratified Datalog

Stratified Datalog

$$Txy \;\text{:–}\; Exy$$
$$Txy \;\text{:–}\; Exz, Tzy$$

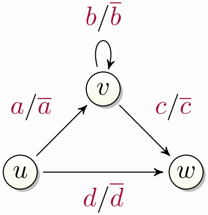
$$Nxy \;\text{:–}\; \neg Txy$$



Stratified Datalog

$$Txy \;:\!-\; Exy$$
$$Txy \;:\!-\; Exz, Tzy$$

$$Nxy \;:\!-\; \neg Txy$$



Negation: can be defined in some semirings

- PosBool: $T^*_{uw} = d \vee (a \wedge c)$

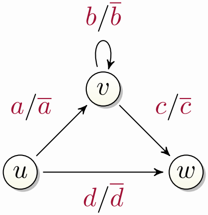
  $N^*_{uw} = \overline{T^*_{uw}} = \overline{d} \wedge (\overline{a} \vee \overline{c})$

Semiring Semantics for Stratified Datalog

Stratified Datalog

$$Txy \; :- \; Exy$$
$$Txy \; :- \; Exz, Tzy$$

$$Nxy \; :- \; \neg Txy$$



Negation: can be defined in some semirings

- PosBool: $T^*_{uw} = d \vee (a \wedge c)$

  $N^*_{uw} = \overline{T^*_{uw}} = \overline{d} \wedge (\overline{a} \vee \overline{c})$

but not clear how do to it in general

- Polynomials: $\overline{a^2} = ?$
- Tropical: $\overline{7} = ?$

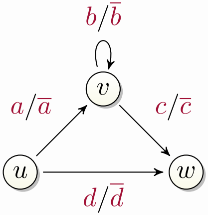
Semiring Semantics for Stratified Datalog

Stratified Datalog

$$Txy \;{:}\!-\; Exy$$

$$Txy \;{:}\!-\; Exz, Tzy$$

$$Nxy \;{:}\!-\; \neg Txy$$

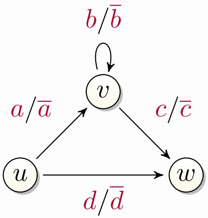


Semiring Semantics for Stratified Datalog

Stratified Datalog

$$Txy \coloneq Exy$$
$$Txy \coloneq Exz, Tzy$$

$$Nxy \coloneq \neg Txy$$



Equation System

$$T_{uv} = a + (a \cdot T_{vv} + d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww} + a \cdot T_{vw})$$

$\rightsquigarrow$

Dualized System

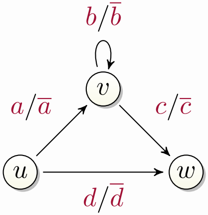
$$N_{uv} = \overline{a} \cdot (\overline{a} + N_{vv}) \cdot (\overline{d} + N_{wv})$$
$$N_{uw} = \overline{d} \cdot (\overline{d} + N_{ww}) \cdot (\overline{a} + N_{vw})$$

Semiring Semantics for Stratified Datalog

Stratified Datalog

$$Txy \;\text{:-}\; Exy$$
$$Txy \;\text{:-}\; Exz, Tzy$$

$$Nxy \;\text{:-}\; \neg Txy$$



Equation System

$$T_{uv} = a + (a \cdot T_{vv} + d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww} + a \cdot T_{vw})$$

$\Longrightarrow$ **Least solution**

$\rightsquigarrow$

Dualized System

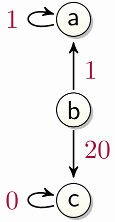
$$N_{uv} = \overline{a} \cdot (\overline{a} + N_{vv}) \cdot (\overline{d} + N_{wv})$$
$$N_{uw} = \overline{d} \cdot (\overline{d} + N_{ww}) \cdot (\overline{a} + N_{vw})$$

$\Longrightarrow$ **Greatest solution**

Motivation II: Fixed-point Logic

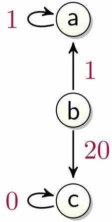
$$\big[\mathbf{gfp}\, Rx.\ \exists y(Exy \wedge Ry)\big](v)$$

"there is an infinite path from v"



Motivation II: Fixed-point Logic

$\left[\mathbf{gfp}\, Rx.\ \exists y(Exy \wedge Ry)\right](v)$ "there is an infinite path from v"



$$R_a = 1 + R_a$$

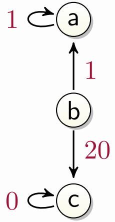
$$R_b = \min(1 + R_a,\ 20 + R_c)$$

$$R_c = 0 + R_c$$

Motivation II: Fixed-point Logic

$$\big[\mathbf{gfp}\, Rx.\ \exists y(Exy \wedge Ry)\big](v)$$

"~~there is~~ cost of an infinite path from v"



$R_a = 1 + R_a$

$R_b = \min(1 + R_a,\ 20 + R_c)$

$R_c = 0 + R_c$

${R_a}^* = \infty$

${R_b}^* = 20$

${R_c}^* = 0$

Greatest Solution

Computing Greatest Fixed Points

Naive Approach

$$
\begin{aligned}
R_a &= 1 + R_a \\
R_b &= \mathsf{min}(1 + R_a,\ 20 + R_c) \\
R_c &= 0 + R_c
\end{aligned}
$$

Naive Approach

Goal: Compute greatest fixed point of a polynomial operator

$$\mathbf{F} : \begin{pmatrix} R_a \\ R_b \\ R_c \end{pmatrix} \mapsto \begin{pmatrix} 1 + R_a \\ \mathsf{min}(1 + R_a, 20 + R_c) \\ 0 + R_c \end{pmatrix}$$

Naive Approach

Goal: Compute greatest fixed point of a polynomial operator

$$\mathbf{F} : \begin{pmatrix} R_a \\ R_b \\ R_c \end{pmatrix} \mapsto \begin{pmatrix} 1 + R_a \\ \min(1 + R_a,\ 20 + R_c) \\ 0 + R_c \end{pmatrix}$$

Iteration:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \mapsto \cdots \mapsto \begin{pmatrix} 20 \\ 20 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 21 \\ 20 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 22 \\ 20 \\ 0 \end{pmatrix} \mapsto \cdots \qquad \begin{pmatrix} \infty \\ 20 \\ 0 \end{pmatrix}$$

Naive Approach

Goal: Compute greatest fixed point of a polynomial operator

$$\mathbf{F} : \begin{pmatrix} R_a \\ R_b \\ R_c \end{pmatrix} \mapsto \begin{pmatrix} 1 + R_a \\ \min(1 + R_a,\ 20 + R_c) \\ 0 + R_c \end{pmatrix}$$

Iteration:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \mapsto \cdots \mapsto \begin{pmatrix} 20 \\ 20 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 21 \\ 20 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 22 \\ 20 \\ 0 \end{pmatrix} \mapsto \cdots \qquad \begin{pmatrix} \infty \\ 20 \\ 0 \end{pmatrix}$$

Naive Approach

Goal: Compute greatest fixed point of a polynomial operator

$$\mathbf{F} : \begin{pmatrix} R_a \\ R_b \\ R_c \end{pmatrix} \mapsto \begin{pmatrix} 1 + R_a \\ \min(1 + R_a, 20 + R_c) \\ 0 + R_c \end{pmatrix}$$

Iteration:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \mapsto \cdots \mapsto \begin{pmatrix} 20 \\ 20 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 21 \\ 20 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 22 \\ 20 \\ 0 \end{pmatrix} \mapsto \cdots \qquad \begin{pmatrix} \infty \\ 20 \\ 0 \end{pmatrix}$$

Faster Computation

Main Result

Let $(K, +, \cdot, 0, 1)$ be an absorptive, fully-continuous semiring.
For a polynomial operator $\mathbf{F}\colon K^n \to K^n$,

$$\mathsf{lfp}(\mathbf{F}) = \mathbf{F}^n(\mathbf{0}), \qquad \mathsf{gfp}(\mathbf{F}) = \mathbf{F}^n\big(\,\mathbf{F}^n(\mathbf{1})^\infty\,\big).$$

Main Result

Let $(K, +, \cdot, 0, 1)$ be an absorptive, fully-continuous semiring.
For a polynomial operator $\mathbf{F}\colon K^n \to K^n$,

$$\mathsf{lfp}(\mathbf{F}) = \mathbf{F}^n(\mathbf{0}), \qquad \mathsf{gfp}(\mathbf{F}) = \mathbf{F}^n\big(\mathbf{F}^n(\mathbf{1})^\infty\big).$$

We only need a **polynomial number** of semiring operations:

$$\underbrace{\begin{pmatrix}0\\0\\0\end{pmatrix} \mapsto \begin{pmatrix}1\\1\\0\end{pmatrix} \mapsto \begin{pmatrix}2\\2\\0\end{pmatrix} \mapsto \begin{pmatrix}3\\3\\0\end{pmatrix}}_{\leq n} \overset{\infty}{\longmapsto} \underbrace{\begin{pmatrix}\infty\\\infty\\0\end{pmatrix} \mapsto \begin{pmatrix}\infty\\20\\0\end{pmatrix} \circlearrowleft}_{\leq n}$$

Which Semirings?

1. **Fully continuous**
   - Natural order: $a \leq a + b$
   - Each chain has supremum $\bigsqcup C$ and infimum $\bigsqcap C$, these commute with $+/\cdot$

Which Semirings?

recall: $(K, +, \cdot, 0, 1)$

1. **Fully continuous**
   - Natural order: $a \leq a + b$
   - Each chain has supremum $\bigsqcup C$ and infimum $\bigsqcap C$, these commute with $+/\cdot$

2. **Absorption**
   - $a + a \cdot b = a$

Which Semirings?

recall: $(K, +, \cdot, 0, 1)$

1 Fully continuous

- Natural order: $a \leq a + b$
- Each chain has supremum $\bigsqcup C$ and infimum $\bigsqcap C$, these commute with $+/\cdot$

2 Absorption

- $a + a \cdot b = a \quad \Leftrightarrow \quad 1$ is greatest element $\quad \Leftrightarrow \quad a \cdot b \leq a$

Which Semirings?

recall: $(K, +, \cdot, 0, 1)$

1 Fully continuous

- Natural order: $a \leq a + b$
- Each chain has supremum $\bigsqcup C$ and infimum $\bigsqcap C$, these commute with $+/\cdot$

2 Absorption

- $a + a \cdot b = a$ $\Leftrightarrow$ 1 is greatest element $\Leftrightarrow$ $\boldsymbol{a \cdot b \leq a}$

Remember:
Decreasing multiplication

Which Semirings?

recall: $(K, +, \cdot, 0, 1)$

1 Fully continuous

- Natural order: $a \leq a + b$
- Each chain has supremum $\bigsqcup C$ and infimum $\bigsqcap C$, these commute with $+/\cdot$

2 Absorption

- $a + a \cdot b = a$ $\Leftrightarrow$ 1 is greatest element $\Leftrightarrow$ $\boldsymbol{a \cdot b \leq a}$

Infinitary Power

For $a \in K$ we define: $a^{\infty} := \bigsqcap_{n<\omega} a^n$

Remember:
Decreasing multiplication

Main Result

Let $(K, +, \cdot, 0, 1)$ be an absorptive, fully-continuous semiring.
For a polynomial operator $\mathbf{F}\colon K^n \to K^n$,

$$\mathsf{lfp}(\mathbf{F}) = \mathbf{F}^n(\mathbf{0}), \qquad \mathsf{gfp}(\mathbf{F}) = \mathbf{F}^n\big(\,\mathbf{F}^n(\mathbf{1})^\infty\,\big).$$

Proof sketch:

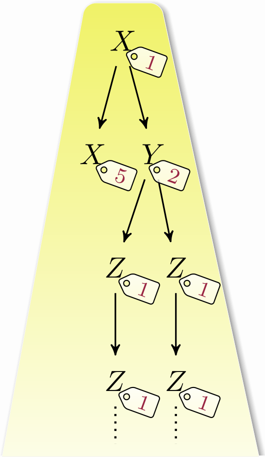
derivation trees

+

absorption

Derivation Trees

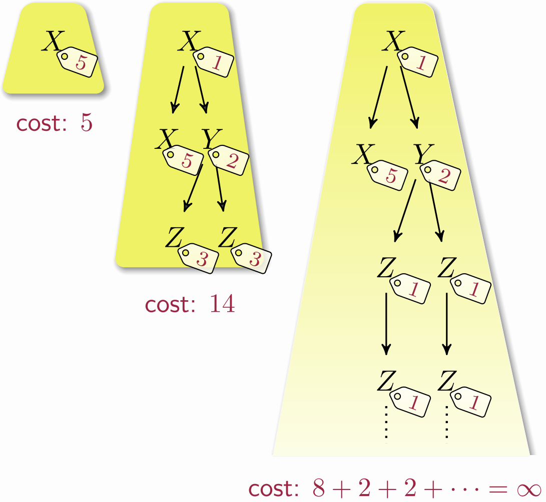
$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \mapsto \begin{pmatrix} \min(5,\ 1+X+Y) \\ 2+Z+Z \\ \min(3,\ 1+Z) \end{pmatrix}$$



cost: $8 + 2 + 2 + \cdots = \infty$

Derivation Trees

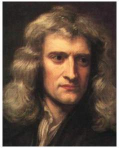
$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \mapsto \begin{pmatrix} \min(5,\ 1+X+Y) \\ 2+Z+Z \\ \min(3,\ 1+Z) \end{pmatrix}$$

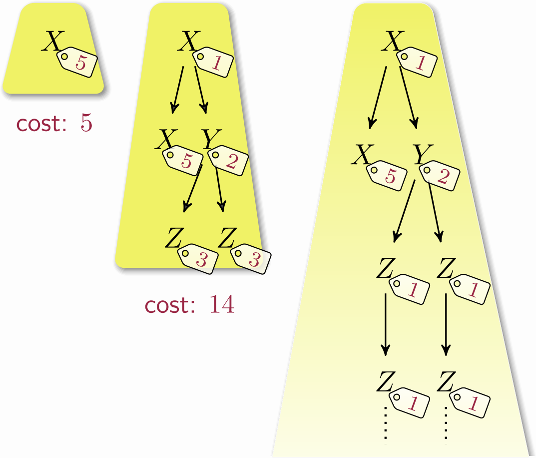


Derivation Trees

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \mapsto \begin{pmatrix} \min(5,\ 1+X+Y) \\ 2+Z+Z \\ \min(3,\ 1+Z) \end{pmatrix}$$

inspired by Newton's method
(Esparza, Kiefer, Luttenberger, JACM'10)





cost: 5

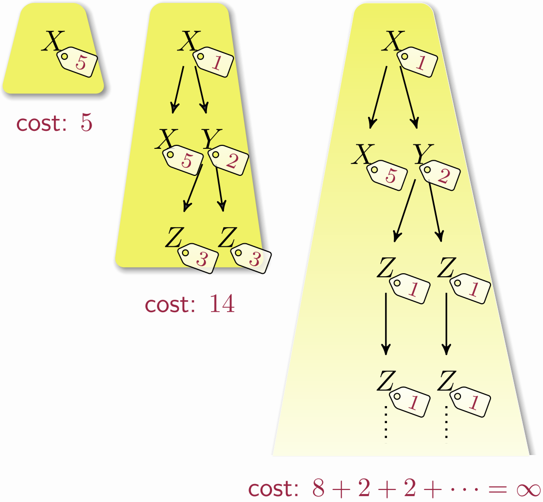
cost: 14

cost: $8+2+2+\cdots=\infty$

Derivation Trees

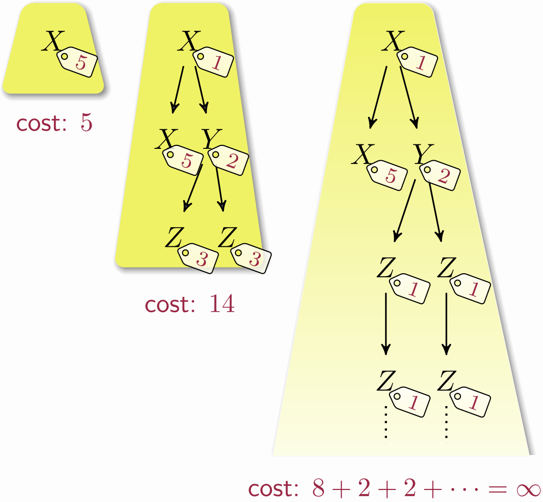
$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \mapsto \begin{pmatrix} \min(5,\ 1+X+Y) \\ 2+Z+Z \\ \min(3,\ 1+Z) \end{pmatrix}$$

inspired by Newton's method
(Esparza, Kiefer, Luttenberger, JACM'10)



Derivation Trees

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \mapsto \begin{pmatrix} \min(5,\ 1+X+Y) \\ 2+Z+Z \\ \min(3,\ 1+Z) \end{pmatrix}$$



inspired by Newton's method
(Esparza, Kiefer, Luttenberger, JACM'10)

lfp $=$ min{cost(🌲) | finite 🌲}

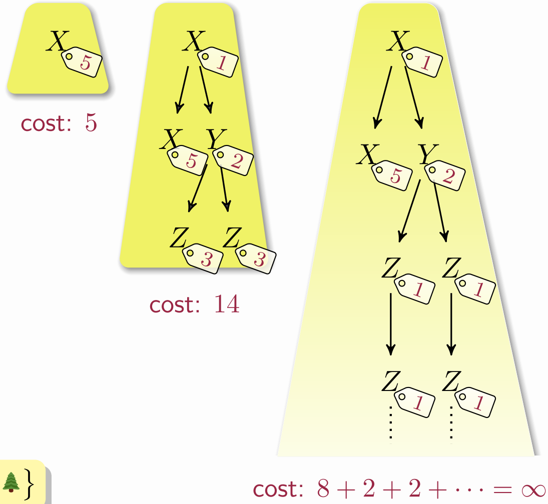
Derivation Trees

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \mapsto \begin{pmatrix} \min(5,\ 1+X+Y) \\ 2+Z+Z \\ \min(3,\ 1+Z) \end{pmatrix}$$

inspired by Newton's method
(Esparza, Kiefer, Luttenberger, JACM'10)

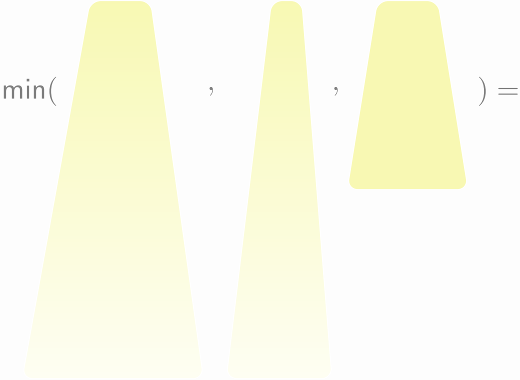
lfp = min{cost(🌲) | finite 🌲}

gfp = min{cost(🌲) | finite 🌲, infinite 🌲}



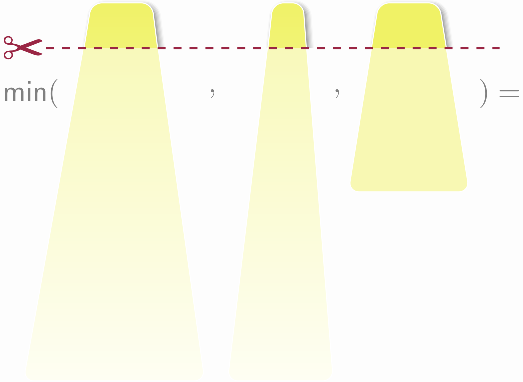
Derivation Trees vs. Iteration

Observation: Prefixes of 🌲 correspond to iteration steps.



Derivation Trees vs. Iteration

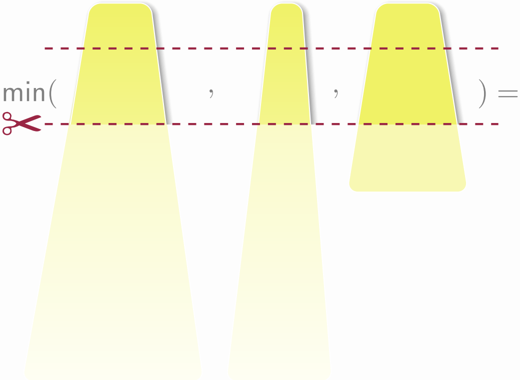
Observation: Prefixes of 🌲 correspond to iteration steps.



$\mathbf{F}(\mathbf{1})$

Derivation Trees vs. Iteration

Observation: Prefixes of 🌲 correspond to iteration steps.

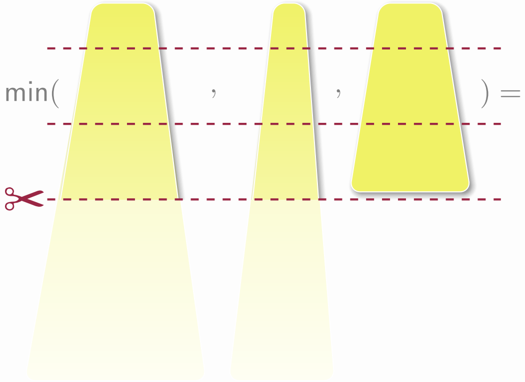


$\mathbf{F}(\mathbf{1})$

$\mathbf{F}^2(\mathbf{1})$

Derivation Trees vs. Iteration

Observation: Prefixes of 🌲 correspond to iteration steps.



$\mathbf{F}(\mathbf{1})$

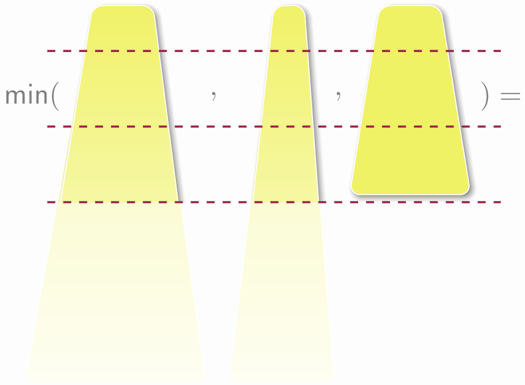
$\mathbf{F}^2(\mathbf{1})$

$\mathbf{F}^3(\mathbf{1})$

$\vdots$

Derivation Trees vs. Iteration

Observation: Prefixes of 🌲 correspond to iteration steps.



$\mathbf{F}(\mathbf{1})$

$\mathbf{F}^2(\mathbf{1})$

$\mathbf{F}^3(\mathbf{1})$

$\vdots$

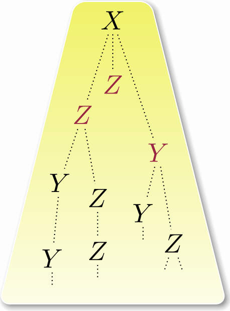
$\bigsqcup_{n<\omega}$: min{cost(🌲) | finite/infinite 🌲} $=$ gfp($\mathbf{F}$) ■

Absorption on Derivation Trees

If each coefficient 🏷 2 occurs more often in 🌳 than in 🌲,
then cost(🌳) is absorbed by cost(🌲).

Absorption on Derivation Trees

If each coefficient 2 occurs more often in 🌳 than in 🌲,
then cost(🌳) is absorbed by cost(🌲).

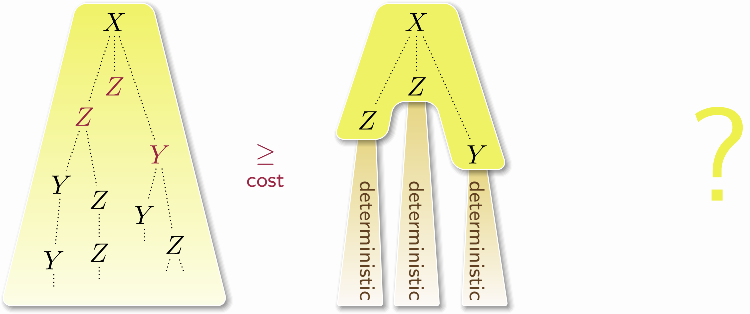


complicated tree 🌳

?

nice tree 🌲

If each coefficient occurs more often in than in ,
then cost() is absorbed by cost().

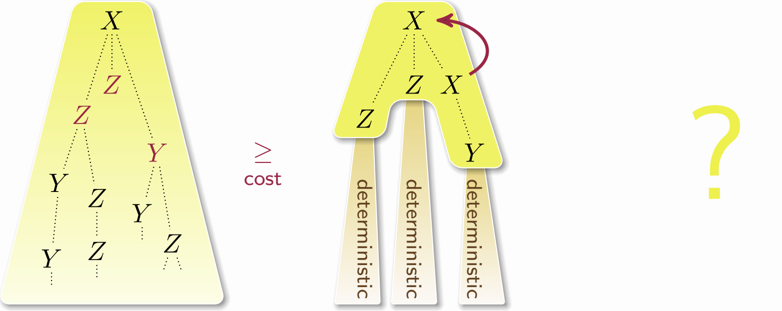


complicated tree

ultimately periodic

nice tree

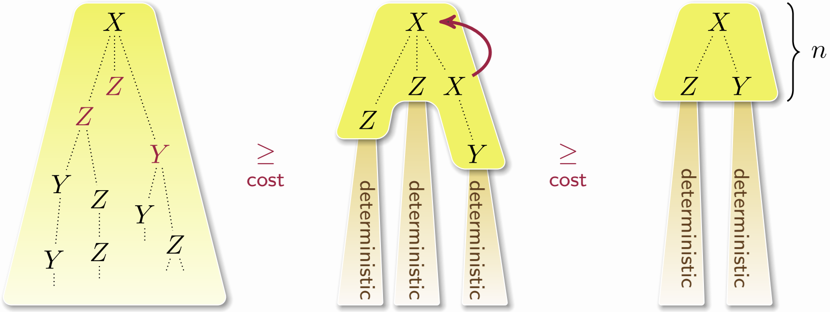
If each coefficient  occurs more often in  than in , then cost() is absorbed by cost().



complicated tree  ultimately periodic nice tree 

Absorption on Derivation Trees

If each coefficient 🏷️2 occurs more often in 🌳 than in 🌲,
then cost(🌳) is absorbed by cost(🌲).

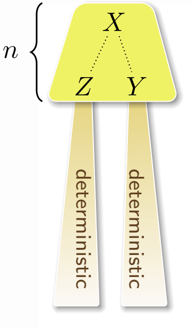


complicated tree 🌳 ultimately periodic nice tree 🌲

Computing Nice Trees

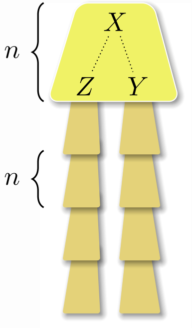
Main Result

$$\mathsf{gfp}(\mathbf{F}) \;=\; \min\left\{\mathsf{cost}(🌲) \;\middle|\; \text{nice } 🌲\right\} \;=\; \ldots$$



Main Result

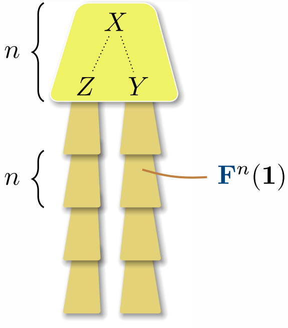
$$\mathrm{gfp}(\mathbf{F}) \;=\; \min\Big\{\mathrm{cost}(🌲) \;\Big|\; \text{nice } 🌲\Big\} \;=\; \ldots$$



Computing Nice Trees

Main Result

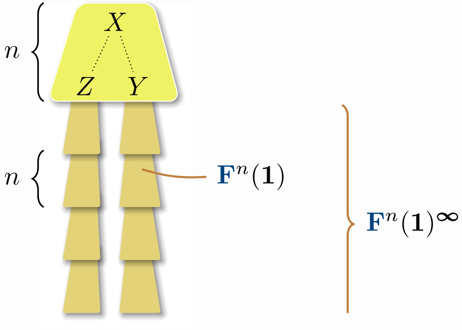
$$\mathrm{gfp}(\mathbf{F}) \;=\; \min\Big\{\mathrm{cost}(🌲) \;\Big|\; \text{nice } 🌲\Big\} \;=\; \ldots$$



Computing Nice Trees

Main Result

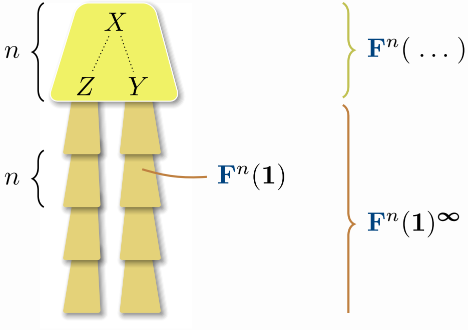
$$\mathsf{gfp}(\mathbf{F}) = \min\{\mathsf{cost}(🌲) \mid \textsf{nice}\ 🌲\} = \ldots$$



Computing Nice Trees

Main Result

$$\mathrm{gfp}(\mathbf{F}) = \min\{\mathrm{cost}(🌲) \mid \text{nice } 🌲\} = \mathbf{F}^n(\mathbf{F}^n(\mathbf{1})^\infty)$$



Back to Datalog: Circuits

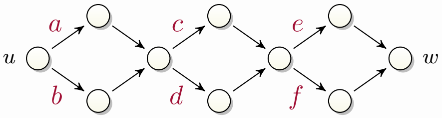
Circuits for Datalog Provenance

Problem: Provenance in polynomial semirings can become large

Datalog

Txy :– Exy

Txy :– Exz, Tzy



PosBool: $T^*_{uw} = ace + acf + ade + adf + bce + bcf + bde + bdf$

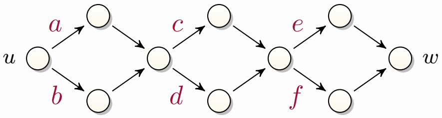
Circuits for Datalog Provenance

Problem: Provenance in polynomial semirings can become large

Datalog

$Txy \;:\!-\; Exy$

$Txy \;:\!-\; Exz, Tzy$



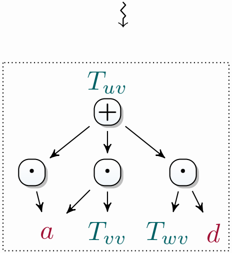
PosBool: $T^*_{uw} = ace + acf + ade + adf + bce + bcf + bde + bdf$

Solution: Represent provenance computation by a small circuit

Circuits for Datalog Provenance

Equation System

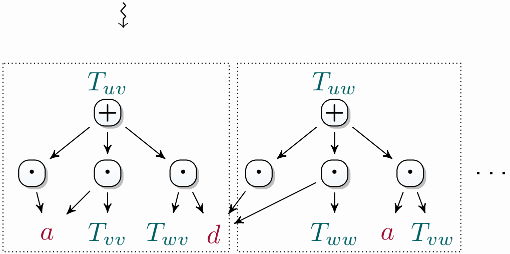
$$T_{uv} = a + (a \cdot T_{vv}) + (d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww}) + (a \cdot T_{vw})$$
$$\vdots$$



Circuits for Datalog Provenance

Equation System

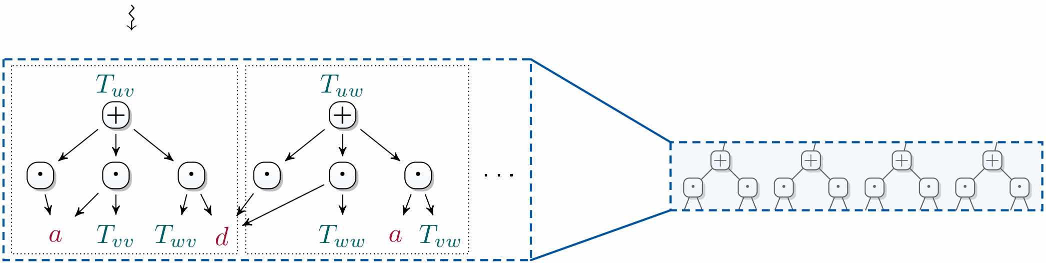
$$T_{uv} = a + (a \cdot T_{vv}) + (d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww}) + (a \cdot T_{vw})$$
$$\vdots$$



Circuits for Datalog Provenance

Equation System

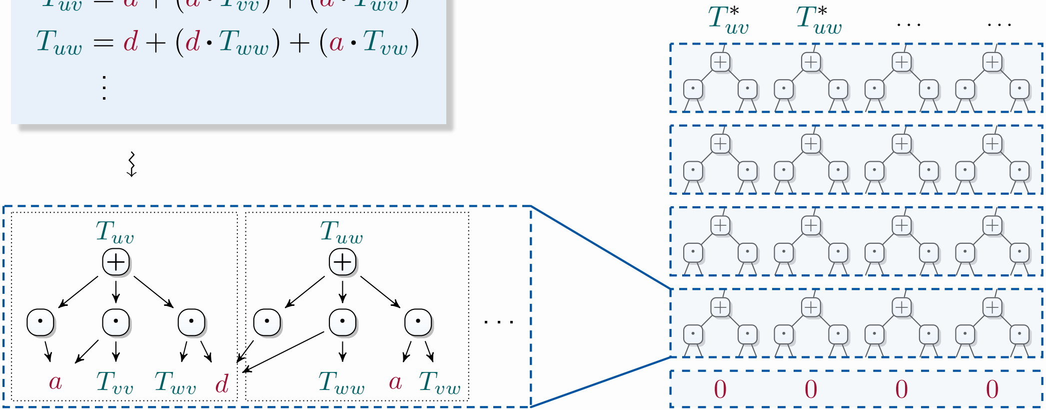
$$T_{uv} = a + (a \cdot T_{vv}) + (d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww}) + (a \cdot T_{vw})$$
$$\vdots$$



Circuits for Datalog Provenance

Equation System

$$T_{uv} = a + (a \cdot T_{vv}) + (d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww}) + (a \cdot T_{vw})$$
$$\vdots$$



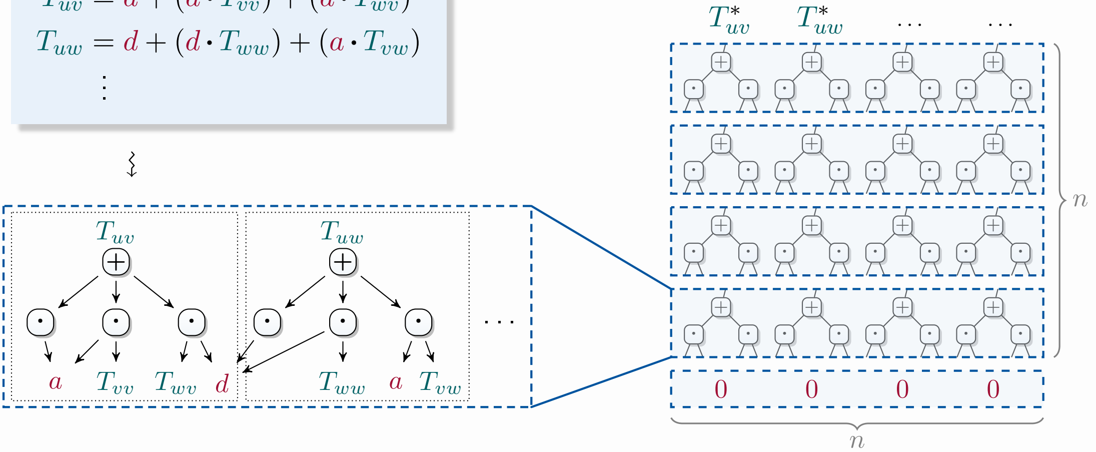
Circuits for Datalog Provenance

Recall

$\mathsf{lfp}(\mathbf{F}) = \mathbf{F}^n(0)$

Equation System

$$T_{uv} = a + (a \cdot T_{vv}) + (d \cdot T_{wv})$$
$$T_{uw} = d + (d \cdot T_{ww}) + (a \cdot T_{vw})$$
$$\vdots$$



Circuits for Stratified Datalog

Strat. Datalog

$$Txy \coloneq Exy$$
$$Txy \coloneq Exz, Tzy$$

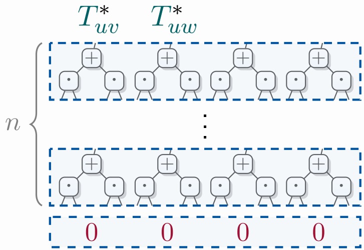
$$Nxy \coloneq \neg Txy$$

Circuits for Stratified Datalog

Strat. Datalog

$$Txy \;\text{:–}\; Exy$$
$$Txy \;\text{:–}\; Exz, Tzy$$

$$Nxy \;\text{:–}\; \neg Txy$$

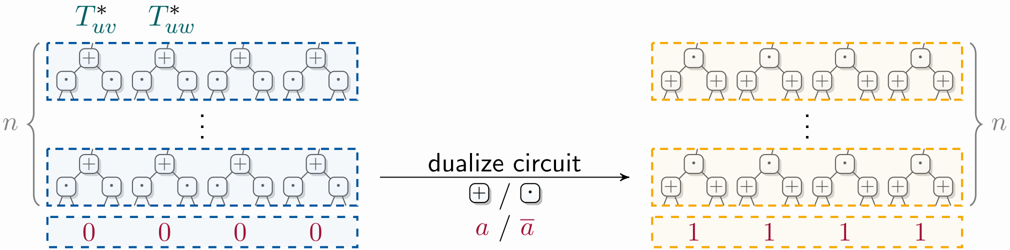


Circuits for Stratified Datalog

Strat. Datalog

$$Txy \text{ :– } Exy$$
$$Txy \text{ :– } Exz, Tzy$$

$$Nxy \text{ :– } \neg Txy$$



Circuits for Stratified Datalog

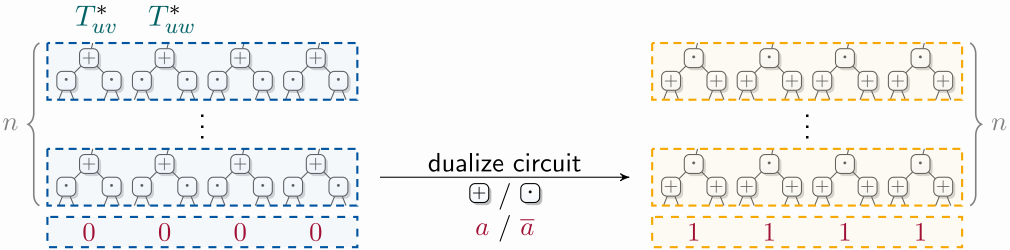
$$\mathrm{gfp}(\mathbf{F}) = \mathbf{F}^n\big(\,\mathbf{F}^n(1)^\infty\,\big)$$

Strat. Datalog

$Txy \;\text{:–}\; Exy$

$Txy \;\text{:–}\; Exz, Tzy$

$Nxy \;\text{:–}\; \neg Txy$



Circuits for Stratified Datalog

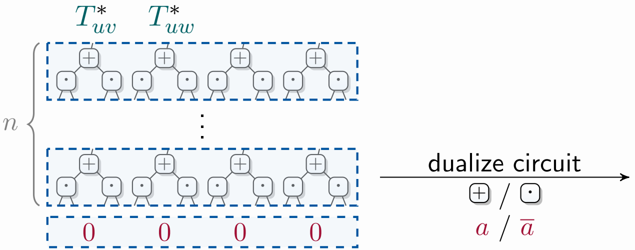
$$\mathsf{gfp}(\mathbf{F}) = \mathbf{F}^n\big(\,\mathbf{F}^n(1)^\infty\,\big)$$

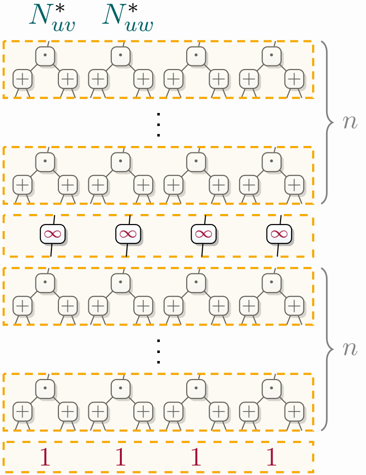
Strat. Datalog

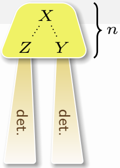
$Txy \;\text{:-}\; Exy$

$Txy \;\text{:-}\; Exz, Tzy$

$Nxy \;\text{:-}\; \neg Txy$





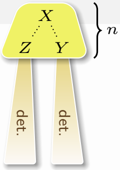


absorption

Computing greatest fixed points

- In absorptive semirings: $\mathrm{gfp}(\mathbf{F}) = \mathbf{F}^n\big(\,\mathbf{F}^n(\mathbf{1})^\infty\,\big)$

Summary



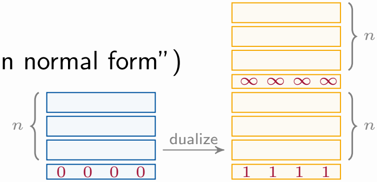
absorption

Computing greatest fixed points

- In absorptive semirings: $\text{gfp}(\mathbf{F}) = \mathbf{F}^n\big(\,\mathbf{F}^n(1)^\infty\,\big)$

Semiring provenance for stratified Datalog

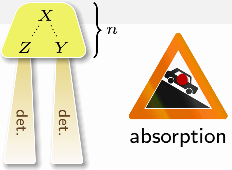
- Negation: greatest solution to dual equation system ("negation normal form")
- Circuit representations for Datalog can be generalized



Summary

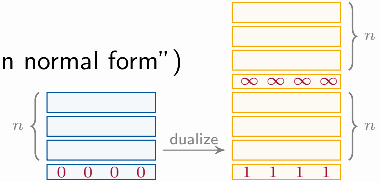
Computing greatest fixed points

- In absorptive semirings: $\mathrm{gfp}(\mathbf{F}) = \mathbf{F}^n\big(\,\mathbf{F}^n(1)^\infty\,\big)$



Semiring provenance for stratified Datalog

- Negation: greatest solution to dual equation system ("negation normal form")
- Circuit representations for Datalog can be generalized



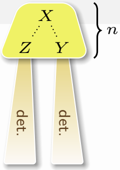
Questions

1. Applications
   - LFP: strategies in infinite games
   - Stratified Datalog: ?

Summary

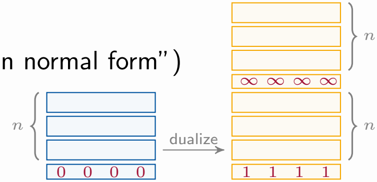
Computing greatest fixed points

- In absorptive semirings: $\text{gfp}(\mathbf{F}) = \mathbf{F}^n\big(\,\mathbf{F}^n(\mathbf{1})^{\infty}\,\big)$



Semiring provenance for stratified Datalog

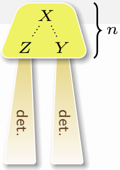
- Negation: greatest solution to dual equation system (“negation normal form”)
- Circuit representations for Datalog can be generalized



Questions

1. Applications
   - LFP: strategies in infinite games
   - Stratified Datalog: ?
2. Alternating fixed points
   - Is the main result applicable ?
   - Quasipolynomial time ?

Summary



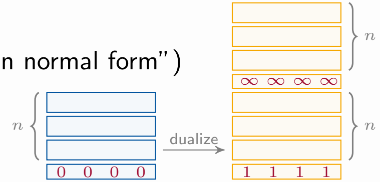
absorption

Computing greatest fixed points

- In absorptive semirings: $\mathrm{gfp}(\mathbf{F}) = \mathbf{F}^n\big(\,\mathbf{F}^n(\mathbf{1})^{\infty}\,\big)$

Semiring provenance for stratified Datalog

- Negation: greatest solution to dual equation system ("negation normal form")
- Circuit representations for Datalog can be generalized



Questions

1. Applications
   - LFP: strategies in infinite games
   - Stratified Datalog: ?
2. Alternating fixed points
   - Is the main result applicable ?
   - Quasipolynomial time ?