Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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SS 2015

Quantum Computing — Assignment 4

Due: Wednesday, 20.05., 14:15

Exercise 1 5 Points

Provide a decomposition of the following transformation into a product of unitary matrices that operate non-trivially only on a two-dimensional subspace of H^4.

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$

Exercise 2 10 Points

(a) Show that no classical gate is universal for quantum computing.

(b) Show that there is no one-qubit universal gate.

(c) Show that for any $d > 2$ there exists a $d \times d$ unitary matrix U which cannot be decomposed as a product of fewer than $d-1$ unitary matrices that operate non-trivially only on a two-dimensional subspace of H^d.

Note that a set Ω of quantum gates is *universal* if any unitary transformation can be approximated to arbitrary precision using only gates from Ω.

Exercise 3 10 Points

Let $\varphi_0 = \alpha\pi$ for some $\alpha \in \mathbb{R} \setminus \mathbb{Q}$, $0 < \alpha < 1$. Show that any gate

$$U_\varphi = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & 1 \end{pmatrix}$$

can be implemented with arbitrary precision ε using $\mathcal{O}(\frac{1}{\varepsilon})$ copies of a single gate U_{φ_0}.

Hint: First show Dirichlet's approximation theorem: For any number $x \in \mathbb{R} \setminus \mathbb{Q}$ and every natural number n, there exist $p, q \in \mathbb{Z}$, $1 \leq q \leq n$, such that

$$0 < |qx - p| \leq \frac{1}{n+1}.$$