Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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WS 2010/11

Mathematical Logic II — Assignment 7

Due: Monday, December 6, 12:00

Exercise 1 3 Points

Let $\tau_1 = \emptyset$ and $\tau_2 = \{c_1, \ldots, c_n\}$ where $c_1, \ldots, c_n$ are constants. Classify all complete theories of the logic $\mathrm{FO}(\tau_i)$ for $i = 1, 2$ using elementary equivalence of τ_i-structures.

Exercise 2 4 Points

Describe four distinct complete extensions of the theory of infinite dense linear orders. Show that there are no further complete extensions of this theory.

Exercise 3 4 + 3 Points

(a) Let $\Phi \subseteq \mathrm{FO}(\tau)$ be a satisfiable set of sentences for some signature τ such that there is an infinite model of Φ. Show that for all $\kappa \in \mathrm{Cn}^\infty$ with $\kappa \geq |\tau|$, Φ has a model of cardinality κ.
Hint: Adjust the proof of the theorem of Löwenheim and Skolem.

(b) Let $\kappa \in \mathrm{Cn}^\infty$. A theory T is *κ-categorical* if it has exactly one model of cardinality κ (up to isomorphism). For a signature τ, let $T \subseteq \mathrm{FO}(\tau)$ be a theory satisfying the following conditions:

   (i) all models of T are infinite;

   (ii) there is some $\kappa \in \mathrm{Cn}^\infty$ with $\kappa \geq |\tau|$ such that T is κ-categorical.

   Show that T is a complete theory.

Exercise 4 3 + 3 Points

Encode the following functions in TA:

(a) $y = 2^x$,

(b) $y = x!$

Hint: Use Gödel's β-function.

Exercise 5* 6* Points

Let $\Phi \subseteq \mathrm{FO}(\tau)$ be a recursively enumerable axiom system for some signature τ. Show that $\Phi^{\models}$ is recursively axiomatizable.
Hint: Find an axiom system Φ' which is equivalent to Φ and whose sentences can be recursively enumerated in a way that their length is strictly increasing.