

Mathematical Logic II — Assignment 3

Due: Monday, November 8, 12:00

Exercise 1

3 + 2 + 4 + 1 Points

- (a) Prove or disprove that there is a history h that contains a limit stage and is a stage itself.
- (b) Are there two histories h and h' that are incomparable with respect to
 - \in ,
 - \subseteq ?
- (c) Let A be a class of histories. What is the least history h such that $\text{acc}(h)$ contains all elements of A ?
- (d) Let H be a class of histories. Is $\bigcap H$ a history?

Exercise 2

4 Points

For a stage s we define $\text{acc}^*(s)$ to be $\text{acc}(s \setminus s')$ if there is a stage s' such that either s' is the maximal limit stage with $s' \in s$ or $s = \mathcal{P}(s')$, and $\text{acc}^*(s) = s$ otherwise. Further define $a_0 = s$ and $a_{n+1} = \text{acc}^*(a_n)$. Prove or disprove that there exists some a_k with $a_k = a_{k+1}$.

Exercise 3

3 Points

Formalise using notions introduced in the lecture and prove that for a set a , $\text{TC}(a) = a \cup \bigcup a \cup \bigcup \bigcup a \cup \dots$

Exercise 4

1 + 1 + 2 + 3 Points

A set a is *inductive* if $\emptyset \in a$ and for all $x \in a$, $x \cup \{x\} \in a$. Let $\omega = \bigcap \{x \mid x \text{ is inductive}\}$.

- (a) Show that ω is a set.
- (b) Is ω inductive?
- (c) Is there an element of ω that is not transitive?
- (d) Prove or disprove that ω is the least limit stage.