Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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Mathematical Logic II — Assignment 2

Due: Tuesday, November 2, 12:00

Exercise 1 4 Points

Let $a \in \mathrm{HF}_n$ for some $n \in \mathbb{N}$. We define $a_0 := a$ and $a_{i+1} = \mathrm{acc}(a_i)$ for $i \in \mathbb{N}$. Prove that there exists some $k \in \mathbb{N}$ with $a_{k+1} = a_k$ and show further that a_k is hereditary and transitive.

Exercise 2 4 Points

Show that the class HF of hereditary finite sets and the class $\mathbb{S} = \{x \mid x = x\}$ of all sets are limit stages.

Exercise 3 5 Points

The cut of a class A is $\mathrm{cut}(A) = \{x \in A \mid\ S(x) \subseteq S(y) \text{ for all } y \in A\ \}$. Let a be a set and $\mathbb{S} = \{x \mid x = x\}$ the class of all sets. Compute $\mathrm{cut}(\mathbb{S})$ and $\mathrm{cut}(\{x \mid a \in x\})$.

Exercise 4 3 + 4 + 6* Points

(a) Every stage is hereditary and transitive. Give a set which is hereditary and transitive, but not a stage.

(b) It follows from the Axiom of Creation that for every set x, the union $\bigcup x = \{z \in S(x) \mid$ there is some $y \in x$ with $z \in y\ \}$ exists. Prove or disprove that the union (the intersection) of a set of stages is a stage. Prove or disprove that the union of a set of histories is a history.

(c)* Consider an arbitrary transitive set x which is linearly ordered by $\in$. A *prefix* of x is a transitive subset of x. Show that a subset $y \subseteq x$ is a prefix of x if and only if $y \in x$ or $y = x$.