Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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WS 2010/11

Mathematical Logic II — Assignment 1

Due: Monday, October 25, 12:00

Exercise 1 4 Points

Show that in any model $(\mathbb{S}, \in)$ of any axiom system of set theory, the class of those sets x that do not have any element y with $x \in y$ is a proper class (not a set). What about the case where $\mathbb{S} = \{\emptyset\}$?

Exercise 2 (2 + 2) + (2 + 4) Points

Recall the definiton of hereditary finite sets: $\mathrm{HF}_0 = \emptyset$ and $\mathrm{HF}_{n+1} = \{x \mid x \subseteq \mathrm{HF}_n\}$.

(a) Prove the following properties of hereditary finite sets.

  (i) $\mathrm{HF}_n \subseteq \mathrm{HF}_{n+1}$ and $\mathrm{HF}_n \in \mathrm{HF}_{n+1}$

  (ii) HF_n has finitely many elements.

(b) Consider the graph $\mathcal{G} = (\mathrm{HF}, E)$ with $E = \{(x, y) \mid x \in y \text{ or } y \in x\}$.

  (i) What is the diameter of $\mathcal{G}$?

  (ii) Show that for all pairwise different $a_1, \ldots, a_n, b_1, \ldots, b_m \in \mathrm{HF}$ there exists a $z \in \mathrm{HF}$ that is connected with all $a_1, \ldots, a_n$, but with no $b_1, \ldots, b_m$ via an edge in $\mathcal{G}$.

Exercise 3 1 + (1 + 1) + 4 Points

(a) Recall the definition of the sets $[n]$ representing natural numbers: $[0] = \emptyset$, $[n + 1] = \{[0], \ldots, [n]\}$. Write the natural number $[4]$ in the set notation (using symbols {, }, $\emptyset$ and commata).

(b) A set x is *transitive* if for all $y \in x$ we have $y \subseteq x$.

  (i) Prove or disprove that a set x is transitive if and only if for all $y \in x$ and all $z \in y$ we have $z \in x$.

  (ii) Prove or disprove that the relation $\in$ on a transitive set is transitive in the usual sense.

(c) Show that every natural number is transitive. Show further that $\in$ is transitive on every natural number and on the set of natural numbers.

Exercise 4* 7* Points

Show that a set is hereditary finite if and only if its transitive closure is finite.