Definition 1. Let A be a class.

- A is *transitive* if $x \in y \in A$ implies $x \in A$.
- A is *hereditary* if $x \subseteq y \in A$ implies $x \in A$.
- $\mathsf{acc}(A) = \{x \mid \text{ there is some } y \in A \text{ such that } x \in y \text{ or } x \subseteq y\}$.

Exercise 1

Prove Lemma 2.2 from the lecture notes.

Lemma 2 . *Let A be a class and b and c sets. The following statements are equivalent:*

(a) $c \in b \in A$ implies $c \in A$, i.e. A is transitive.

(b) $b \in A$ implies $b \subseteq A$.

(c) $b \in A$ implies $b \cap A = b$.

Exercise 2

Prove Lemma 2.3 from the lecture notes.

Lemma 3 . *Let A and B be classes.*

(a) If B is hereditary transitive and $A \subseteq B$ then $\mathsf{acc}(A) \subseteq B$.

(b) A is hereditary transitive if and only if $\mathsf{acc}(A) = A$.

Exercise 3

Let a, b be sets and A, B be proper nonempty classes. Let φ be a property. Which of the following classes are sets: $a \cap B$, $\{x \in B \mid x \in a, \varphi\}$, $\{x \in b \mid x \in A, \varphi\}$, $a \setminus B$, $A \cap B$, $\bigcap B$?

Exercise 4

It follows from the Axiom of Creation that for every set x there exists a transitive set y with $x \subseteq y$. Show that then there is an unambigous *least* transitive set $\mathsf{TC}(x)$ such that $x \subseteq \mathsf{TC}(x)$ holds. (TC = Transitive Closure)
Hint Intuitively, $\mathsf{TC}(x) = x \cup \bigcup x \cup \bigcup\bigcup x \ldots$