Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, K. Dannert

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Logic and Games — Assignment 4

Due: Tuesday the 13th November at 12:00 in the lecture or at our chair.

Exercise 1 — 4 Points

Let $\mathcal{G} = (V, V_0, V_1, E, \Omega)$ be a parity game with winning regions W_0 and W_1 and let f be a positional strategy for Player 0. Prove or disprove the following statements:

(a) If f is a winning strategy for Player 0 on W_0 then $f(V_0 \cap W_0) \subseteq W_0$.

(b) If $f(V_0 \cap W_0) \subseteq W_0$ then f is a winning strategy for Player 0 on W_0.

Exercise 2 — 8 Points

A parity game $\mathcal{G} = (V, V_0, V_1, E, \Omega)$ is called *weak*, if $\Omega(v) \leq \Omega(w)$ for every edge $(v, w) \in E$.

(a) Let $m = \max(\Omega(V))$ and $V_m = \{v \in V : \Omega(v) = m\}$ the set of positions with the maximum priority. Prove that in a weak parity game the set $\mathrm{Attr}_\sigma(V_m)$ is a trap for Player $1-\sigma$. Does this also hold for general parity games?

(b) Give a polynomial time algorithm which computes the winning regions in weak parity games.

Exercise 3 — 6 Points

Give a polynomial time algorithm which computes the winning regions of parity games on *undirected* trees.

Exercise 4 — 12 Points

A *Büchi-Game* $\mathcal{G} = (V, V_0, V_1, E, F)$, with $F \subseteq V$, is a game in which Player 0 wins an infinite play if and only if nodes from F are visited infinitely often. We say that Player 1 plays with a *coBüchi* winning condition in this game.

(a) Make precise and prove the statement that Büchi/coBüchi-games are special cases of parity games.

(b) Give an algorithm which computes the winning regions of both players in a Büchi-game in polynomial time.