Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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WS 2018/19

Logic and Games — Assignment 2

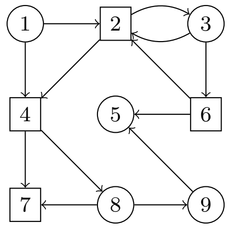
Due: 23th October at 12:00 in the lecture or at our chair.

Exercise 1 10 Points

(a) Construct the satisfiability game $\mathcal{G}_\psi$ for the given Horn formula ψ and compute the winning regions of both players.

$$(U \to Y) \land (Y \land Z \to V) \land (1 \to U) \land (X \to Z) \land (U \land Y \to X) \land (V \to 0)$$

(b) Construct the Horn formula $\psi_\mathcal{G}$ for the following game graph $\mathcal{G}$ and determine, using the marking algorithm for Horn formulae, whether Player 0 wins from node 1.



Exercise 2 13 Points

A *threshold game* $\mathcal{G} := (V, E, t)$ consists of a finite directed graph (V, E) and a threshold function $t\colon V \to \mathbb{N}_0$. From position $v \in V$ the rules are as follows:

1. Player 0 chooses a set $X \subseteq vE$ with $|X| \geq t(v)$.
2. Player 1 chooses a node $v' \in X$. The play continues from v'.

The first player who is unable to move loses. Infinite plays are draws.
The decision problem whether Player 0 has a winning strategy from a given node is defined as

$$\textsc{Threshold} := \left\{ (\mathcal{G}, v) \ : \ \mathcal{G} \text{ is a threshold game}, v \in W^0_\mathcal{G} \right\}.$$

(a) Show that Threshold is Ptime-hard, by reducing Game in Logspace to Threshold, i.e. showing Game $\leq_{\textsc{Logspace}}$ Threshold.[1]

(b) Show that Threshold $\in$ Ptime, by proving Threshold $\leq_{\textsc{Ptime}}$ Sat-Horn.

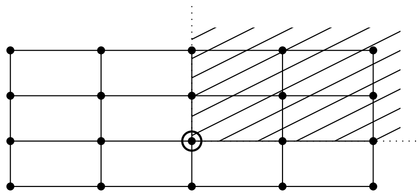
(c) Provide (in pseudo-code) an *alternating* Logspace-algorithm that decides Threshold.

[1] As in the lecture, Game denotes the decision problem for reachability games whether the given node is in the winning region of player 0.

Exercise 3 7 Points

A rectangular chocolate bar with $n \times m$ pieces can be seen as a $\{0, \ldots, n\} \times \{0, \ldots, m\}$ grid, such that the faces of the grid correspond to the pieces and the edges to the break lines between the pieces.

Consider the following two player game on a chocolate bar: The players alternate each turn, in which the current player chooses a node of the corresponding grid, that is the bottom left corner of a still existing piece. All pieces to the top right of the node are removed. Whoever takes the last piece loses.



Show that one player (who?) has a winning strategy for each size of the bar (except for one special case).

Hint: Do not present the strategy, but rather prove its existence using the determinacy theorem for finite, well-founded[2] reachability games.

[2]A game is well-founded it does not admit infinite plays.