Lehr- und Forschungsgebiet
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SS 2017

Logics for Reasoning about Uncertainty — Exercise sheet 2

Due: Tuesday, 30 May.

Note: The exercises for this lecture are voluntary. To have a solution corrected, you may hand it in until the date specified on the exercise sheet.

Exercise 1

We consider the modal logic KD45 with the axioms (K), (4), (5) together with (D): $\neg K_a 0$. We know that this characterizes the Kripke frames that are serial, Euclidean, and transitive.

a) Prove that every modal formula ψ with only one agent (i. e. $|A| = 1$) that is satisfiable in a serial, Euclidean and transitive Kripke structure is also satisfiable in a Kripke structure over a frame (W, E) where $W = \{s\} \cup W'$ and $E = W \times W'$.

b) Prove that Sat(KD45) for $|A| = 1$ is NP–complete.

Exercise 2

The additional axioms and rules for incorporating common knowledge are:

- $C1$: $E_G\psi \rightarrow \bigwedge_{a\in G} K_a\psi$
- $C2$: $C_G\psi \rightarrow E_G(\psi \wedge C_G\psi)$
- RC: From $\psi \rightarrow E_G(\psi \wedge \varphi)$ infer $\psi \rightarrow C_G\varphi$

Prove that this axiomatization is sound.

Exercise 3

First-order logic of knowledge is obtained by closing first–order logic under knowledge operators K_a for $a \in A$. That is, the syntax of first order logic is extended by the rule: If ψ is a formula, then so is $K_a\psi$.

A relational Kripke structure of vocabulary τ is the extension of a Kripke frame $(W, (E_a)_{a\in A})$ by a function that assigns to each world $w \in W$ a τ–structure A_w.

The common domain assumption imposes that all structures A_w have the same universe. The weaker domain inclusion assumption assumes that whenever world w is considered possible at world v then the universe of A_v should be contained in the universe of A_w.

How would you formally define the semantics of first-order logic of knowledge in relational Kripke structures? What is the rule of the domain assumptions? How should valuations of variables be defined? The intention is that knowledge of equality and knowledge of inequality should hold, i.e. $x = y \rightarrow K_a(x = y)$ and similarly for $\neq$.

The Barcan formula is the implication $\forall x_1 \ldots \forall x_k K_a\psi \rightarrow K_a\forall x_1 \ldots \forall x_k\psi$.

a) Show that the Barcan formula is valid under the common domain assumption, but not under the domain inclusion assumption

b) What about the converse of the Barcan formula?