

Logics for Reasoning about Uncertainty — Exercise sheet 1

Due: Tuesday, 16 May.

Note: The exercises for this lecture are voluntary. To have a solution corrected, you may hand it in until the date specified on the exercise sheet.

Exercise 1

Consider the variant of the muddy children puzzle where the father initially says: “Tom has mud on his forehead”. Show that the children, other than Tom, are unable to deduce whether they are muddy or not. Explain why this is the case despite the fact that they apparently have more information than in the standard variant of the puzzle.

Exercise 2

Prove that the following formulae are valid for knowledge structures (assuming that the relations E_a are equivalence relations):

- $D_G\varphi \rightarrow D_G D_g\varphi$
- $\neg D_G\varphi \rightarrow D_G\neg D_G\varphi$
- $\neg\varphi \rightarrow K_a\neg K_a\varphi$
- $\neg\varphi \rightarrow K_1 K_2 \dots K_i \neg K_i \dots K_2 K_1 \varphi$
- $\neg K_a \neg K_a \varphi \leftrightarrow K_a \varphi$

Exercise 3

Is the following deduction rule sound (i. e. does it preserve validity)?

From

$$\left(\bigwedge_{a \in G} \psi_a \right) \rightarrow \varphi$$

infer

$$\left(\bigwedge_{a \in G} K_a \psi_a \right) \rightarrow D_G \varphi.$$