Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, D. Fischer, T. Ganzow,
B. Puchala

WS 09/10

Complexity Theory and Quantum Computing — Assignment 8

Due: Monday, December 21, 12:00

Exercise 1

Prove the following claims:

(a) $\mathrm{NP}^{\mathrm{BPP}} \subseteq \mathrm{BPP}^{\mathrm{NP}}$.

Hint: Show that the language consisting of all words $x\#r$ such that x is accepted by a polynomial NTM M using a BPP oracle with random bit-sequence r is in NP. Consider, for a fixed sequence of random bits, the number of oracle queries for which the BPP oracle gives the wrong answer.

(b) $\mathrm{BPP}^{\mathrm{BPP}} = \mathrm{BPP}$.

Hint: Consider a suitable simulation of the oracle machine.

(c) $\mathrm{NP} \subseteq \mathrm{BPP}$ implies $\Sigma_2^p \subseteq \mathrm{BPP}$. Show that this implies $\mathrm{PH} \subseteq \mathrm{BPP}$.

Exercise 2

A non-deterministic Turing machine M is called *standardised* if it has, for each configuration, exactly two possible successor configurations, and if there is a polynomial p such that each computation path has a length of exactly $p(n)$ (where n is the length of the input). Hence, the computation tree of such a machine M on an input x of length n is a full binary tree of depth n with $2^{p(n)}$ leaves. Each leaf is labelled by 0 or 1 to denote a rejecting or accepting state, respectively. If you read the labels from left to right, you obtain the word $M(x) \in \{0,1\}^{2^{p(n)}}$.

We introduce the following uniform model for defining complexity classes: Let $A, R \subseteq \{0,1\}^*$ be a pair of disjoint so-called *leaf languages*. Then (A, R) defines the complexity class $\mathcal{C}[A, R]$ of those languages for which there exists a standardised non-deterministic Turing machine M such that $x \in L$ if, and only if, $M(x) \in A$ and $x \notin L$ if, and only if, $M(x) \in R$.

(a) Specify pairs (A, R) of leaf-languages such that $\mathcal{C}[A, R]$ corresponds to the following complexity classes: P, NP, NP ∩ coNP, RP, coRP, ZPP, BPP, Σ_2^p, PSPACE.

Hint: Use the characterisation of classes in the Polynomial Hierarchy by alternating Turing machines.

(b) For which of these classes can A and R be chosen such that $R = \bar{A}$?

(c) Prove that $\mathcal{C}[A, R] = $ PSPACE if A is NLOGSPACE-complete.

Exercise 3

Prove the following closure properties of probabilistic complexity classes:

(a) PP, BPP, RP and ZPP are closed under polynomial-time reductions.

(b) BPP and RP are closed under union and intersection.

(c) PP is closed under complement and symmetric difference.

Hint: Eliminate the asymmetry in the definition concerning the acceptance probability of $1/2$. Show that, for every probabilistic TM M, there exists a PTM M' such that, for all inputs x, $\Pr[M' \text{ acc. } x] \neq 1/2$ and $\Pr[M' \text{ acc. } x] > 1/2$ if, and only if, $\Pr[M \text{ acc. } x] > 1/2$.