Due: Monday, December 14, 12:00

Exercise 1
Let GEN be the problem from Assignment 4, Exercise 1, (b). Construct an explicit ALOGSPACE-algorithm for GEN.

Exercise 2
An ATM $M$ with adress tape uses one of the working tapes to adress cells of the input tape. If $M$ writes the number $i$ in binary representation to the adress tape, then the head on the input tape goes to cell number $i$ in one step. This means, that $M$ can read arbitrary bits of the input in logarithmic time. Therefore, it makes sense to define complexity classes $\text{Atime}(T(n))$ for $T(n) < n$.

(a) Give a precise definition of this machine model.

(b) Prove that $\text{PAL} \in \text{Alogtime} = \text{Atime}(O(\log n))$, where PAL is the problem from Assignment 2, Exercise 3.

Exercise 3
Let $A$, $B$ and $C$ be $n \times n$ matrices. The decision problem whether $AB = C$ can obviously be solved in time $O(n^3)$. This trivial bound can be improved, however, no deterministic algorithm is known which solves this problem in time less than $O(n^{2.3})$. Now consider the following randomized algorithm.

\begin{verbatim}
input: A, B, C
choose a vector $x \in \{-1, 1\}^n$ at random
if $A(Bx) \neq Cx$: reject, else: accept.
\end{verbatim}

Prove that this algorithm solves the problem with $O(n^2)$ arithmetical operations and with an error probability less than $\frac{1}{2}$.

Exercise 4
Prove the following fact: If $\text{NP} \subseteq \text{BPP}$, then $\text{RP} = \text{NP}$.

Hint: First, prove that from a $\text{Ptime}$ algorithm for SAT one can construct a $\text{Ptime}$ algorithm which constructs, for a given satisfiable formula, a model of this formula. Now use the fact that from $\text{NP} \subseteq \text{BPP}$ in particular it follows that SAT $\in$ BPP.

http://www.logic.rwth-aachen.de/Teaching/KTQC-WS09/