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WS 09/10

Complexity Theory and Quantum Computing — Assignment 7

Due: Monday, December 14, 12:00

Exercise 1

Let GEN be the problem from Assignment 4, Exercise 1, (b). Construct an explicit ALogspace-algorithm for GEN.

Exercise 2

An ATM M with adress tape uses one of the working tapes to adress cells of the input tape. If M writes the number i in binary representation to the adress tape, then the head on the input tape goes to cell number i in one step. This means, that M can read arbitrary bits of the input in logarithmic time. Therefore, it makes sense to define complexity classes Atime($T(n)$) for $T(n) < n$.

(a) Give a precise definition of this machine model.

(b) Prove that PAL $\in$ ALogtime = Atime($O(\log n)$), where PAL is the problem from Assignment 2, Exercise 3.

Exercise 3

Let A, B and C be $n \times n$ matrices. The decision problem whether $AB = C$ can obviously be solved in time $O(n^3)$. This trivial bound can be improved, however, no deterministic algorithm is known whichs solves this problem in time less than $O(n^{2.3})$. Now consider the following randomized algorithm.

input: A, B, C
choose a vector $x \in \{-1, 1\}^n$ at random
if $A(Bx) \neq Cx$: reject, **else**: accept.

Prove that this algorithm solves the problem with $O(n^2)$ arithmetical operations and with an error probability less than $\frac{1}{2}$.

Exercise 4

Prove the following fact: If NP $\subseteq$ BPP, then RP = NP.
Hint: First, prove that from a Ptime algorithm for SAT one can construct a Ptime algorithm which constructs, for a given satisfiable formula, a model of this formula. Now use the fact that from NP $\subseteq$ BPP in particular it follows that SAT $\in$ BPP.