Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, D. Fischer, T. Ganzow,
B. Puchala

WS 09/10

Complexity Theory and Quantum Computing — Assignment 5

Due: Monday, November 30, 12:00

Exercise 1

(a) Let AGEN be the variant of the problem GEN from Assignment 4, Exercise 1, where the function $\circ$ is associative. Prove that AGEN is in NLOGSPACE.

(b) An undirected graph $G = (V, E)$ is called k-colourable for a natural number k if there is a function $f : V \to \{1, \ldots, k\}$ such that $f(u) \neq f(v)$ for all $(u, v) \in E$. The problem k-colourability asks, given a graph $G = (V, E)$, whether G is k-colourable. It is known that the problem 3-colourability is NP-complete. Determine the complexity of the problem 2-colourability.

Exercise 2

A homomorphism from a graph $G = (V_G, E_G)$ to a graph $H = (V_H, E_H)$ is a function $f : V_G \to V_H$ such that for all $(u, v) \in E_G$ we also have $(f(u), f(v)) \in E_H$. The graph homomorphism problem asks, given two undirected graphs G and H, whether there is a homomorphism from G to H.

(a) Prove that the graph homomorphism problem is NP-complete.

(b) Analyse the complexity of the graph homomorphism problem in the case where G is fixed and in the case where H is fixed.

Hint: Consider graph colourability.

Exercise 3

The game Geography is played by two players on a directed graph $G = (V, E)$ with a distinguished starting position u. The first player starts at position u. Then, the players move to a successor position $w \in vE$ of the last position v in alternation. They are only permitted to choose positions that have not been visited before. If a player has no legal move, he loses. GEOGRAPHY is the problem to decide, for a given graph G and a position u, whether player 0 has a winning strategy for Geography on G from u. Prove that GEOGRAPHY is PSPACE-complete.
Hint: Use a reduction of QBF to GEOGRAPHY.