Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, D. Fischer, T. Ganzow,
B. Puchala

WS 09/10

Complexity Theory and Quantum Computing — Assignment 4

Due: Monday, November 23, 12:00

Exercise 1

(a) HORN-3SAT is the satisfiability problem for formulae $\psi = \bigwedge_i \bigvee_j Y_{ij}$ in CNF, that consist of Horn-clauses containing at most three literals each. Show that HORN-3SAT is P-complete (with respect to logspace-reductions).

(b) Let $A = \{1, \dots, n\}$ be a nonempty set, $\circ$ a binary function on A and S a subset of A. The closure $\langle S \rangle$ of S in A is the smallest subset $U \subseteq A$ with $S \subseteq U$ such that U is closed under $\circ$, i.e., if $u, v \in U$, then $u \circ v \in U$.
The problem GEN asks, given A, $\circ$, S and $c \in A$, whether $c \in \langle S \rangle$. Prove that GEN is P-complete.
Hint: Prove that GEN $\in$ P and HORN-3SAT $\leq_{\log}$ GEN.

Exercise 2

Prove that the class $\textsc{Polylogspace} = \bigcup_{d \in \mathbb{N}} \textsc{Dspace}((\log n)^d)$ has no complete problems with respect to logspace reductions and that the class $\textsc{Ptime}$ has no complete problems with respect to linear time reductions.

Exercise 3

Prove the following facts.

(a) A language $L \subseteq \Sigma^*$ is NP-complete if, and only if, its complement $\overline{L} = \Sigma^* \setminus L$ is co-NP-complete.

(b) $\mathrm{P} \neq \textsc{Dspace}(n)$.

Exercise 4

Prove the following facts.

(a) NP = coNP if, and only if, there are (NP $\cup$ coNP)-complete problems.

(b) $L \in$ NP $\cap$ coNP if, and only if, L is decidable by an error-free polynomial-time bounded nondeterministic TM.