Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, D. Fischer, T. Ganzow,
B. Puchala

WS 09/10

Complexity Theory and Quantum Computing — Assignment 3

Due: Monday, November 16, 12:00

Exercise 1

A function $t : \mathbb{N} \to \mathbb{N}$ is called time constructible, if there is a Turing machine M such that $\text{Time}_M(x) = t(|x|)$ for each input x. Analogously, a function $s : \mathbb{N} \to \mathbb{N}$ is called space constructible, if there is a Turing machine M such that $\text{Space}_M(x) = s(|x|)$ for each input x. Prove the following properties of time constructible functions.

(a) For each computable function $f : \mathbb{N} \to \mathbb{N}$ there is a time constructible function t such that $t(n) > f(n)$ for all $n \in \mathbb{N}$.

(b) Each time constructible function is space constructible as well, but the converse is not true.

Exercise 2

Determine the order of the following complexity classes with respect to the subset relation: $\text{DTIME}(n^3)$, $\text{DTIME}(n!)$, P, LOGSPACE, $\text{DSPACE}(\log^2 n)$, $\text{DSPACE}(n \log n)$. Prove proper inclusions if possible.

Exercise 3

A language $L \subseteq \Sigma^*$ is called context sensitive if it can be generated by a context sensitive grammar. A grammar G over an alphabet $\Gamma \supset \Sigma$ is a finite set of rules of the form $w \to w'$ with $w, w' \in \Gamma^*$. Moreover, there is a distinguished initial symbol $S \in \Gamma \setminus \Sigma$. G is called context sensitive if, for each rule $w \to w'$ of G, we have $|w| \leq |w'|$ (except for the rule $S \to \varepsilon$ if it is contained in G). A rule $w \to w'$ enables precisely the derivations of the form $uwv \to uw'v$ for any $u, v \in \Gamma^*$. The language generated by G in Σ^* is the set

$$L(G) = \{w \in \Sigma^* \mid w \text{ can be derived from } S \text{ by a finite sequence of rules from } G\}.$$

Finally, we define $\text{CSL} := \{L \mid L \text{ is context sensitive}\}$. Prove that $\text{CSL} \subseteq \text{NSPACE}(n)$.

Exercise 4

Prove that $\text{NSPACE}(n) \subseteq \text{CSL}$.
Hint: A language in $\text{NSPACE}(n)$ can be decided by a nondeterministic Turing machine with a single tape and with a unique accepting configuration given by state q^+, head position 0 and empty tape. Then, a configuration of this Turing machine can be represented as a word $w_0 \dots w_{p-1}(qw_p)w_{p+1} \dots$ over the alphabet $\Sigma \cup (Q \times \Sigma)$.
Remark: The result $\text{NSPACE}(n) = \text{CSL}$ shows that the class of all context sensitive languages is closed under complement.