Complexity Theory and Quantum Computing — Assignment 3

Due: Monday, November 16, 12:00

Exercise 1
A function $t : \mathbb{N} \to \mathbb{N}$ is called time constructible, if there is a Turing machine $M$ such that $\text{Time}_M(x) = t(|x|)$ for each input $x$. Analogously, a function $s : \mathbb{N} \to \mathbb{N}$ is called space constructible, if there is a Turing machine $M$ such that $\text{Space}_M(x) = s(|x|)$ for each input $x$. Prove the following properties of time constructible functions.

(a) For each computable function $f : \mathbb{N} \to \mathbb{N}$ there is a time constructible function $t$ such that $t(n) > f(n)$ for all $n \in \mathbb{N}$.

(b) Each time constructible function is space constructible as well, but the converse is not true.

Exercise 2
Determine the order of the following complexity classes with respect to the subset relation: $\text{DTIME}(n^3)$, $\text{DTIME}(n!)$, $\mathcal{P}$, $\mathcal{L}$, $\text{DSPACE}(\log^2 n)$, $\text{DSPACE}(n \log n)$. Prove proper inclusions if possible.

Exercise 3
A language $L \subseteq \Sigma^*$ is called context sensitive if it can be generated by a context sensitive grammar. A grammar $G$ over an alphabet $\Gamma \supset \Sigma$ is a finite set of rules of the form $w \rightarrow w'$ with $w, w' \in \Gamma^*$. Moreover, there is a distinguished initial symbol $S \in \Gamma \setminus \Sigma$. $G$ is called context sensitive if, for each rule $w \rightarrow w'$ of $G$, we have $|w| \leq |w'|$ (except for the rule $S \rightarrow \varepsilon$ if it is contained in $G$). A rule $w \rightarrow w'$ enables precisely the derivations of the form $uwv \rightarrow uw'v$ for any $u, v \in \Gamma^*$. The language generated by $G$ in $\Sigma^*$ is the set

$L(G) = \{ w \in \Sigma^* | w \text{ can be derived from } S \text{ by a finite sequence of rules from } G \}$

Finally, we define CSL := $\{ L | L \text{ is context sensitive} \}$. Prove that CSL $\subseteq$ NSPACE($n$).

Exercise 4
Prove that $\text{NSPACE}(n) \subseteq \text{CSL}$.

Hint: A language in $\text{NSPACE}(n)$ can be decided by a nondeterministic Turing machine with a single tape and with a unique accepting configuration given by state $q^+$, head position 0 and empty tape. Then, a configuration of this Turing machine can be represented as a word $w_0 \ldots w_{p-1}(qw_p)w_{p+1} \ldots$ over the alphabet $\Sigma \cup (Q \times \Sigma)$.

Remark: The result $\text{NSPACE}(n) = \text{CSL}$ shows that the class of all context sensitive languages is closed under complement.

http://www.logic.rwth-aachen.de/Teaching/KTQC-WS09/