Lehr- und Forschungsgebiet
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WS 09/10

Complexity Theory and Quantum Computing — Assignment 2

Due: Monday, November 9, 12:00

Exercise 1

For $A \subseteq \mathbb{N}$ we consider the unary representation $\mathrm{UN}(A) = \{1^n : n \in A\}$ and the binary representation $\mathrm{BIN}(A) = \{\mathrm{bin}(n) : n \in A\}$ of A. Prove the following facts.

(a) $\mathrm{UN}(A) \in \mathrm{P}$ if and only if $\mathrm{BIN}(A) \in \textsc{Dtime}(2^{O(n)})$.

(b) $\mathrm{UN}(A) \in \bigcup_{d\in\mathbb{N}} \textsc{Dspace}(\log^d(n))$ if and only if $\mathrm{BIN}(A) \in \textsc{Pspace}$.

Exercise 2

(a) It is known that $\textsc{Dspace}(0) = \mathrm{REG}$, i.e., Turing machines that do not write to the working tape recognize precisely the regular languages. Use this fact to prove that $\textsc{Dspace}(O(1)) = \mathrm{REG}$.

(b) Let $L = \{\mathrm{bin}(1)\#\mathrm{bin}(2)\#\dots\#\mathrm{bin}(k) : k \in \mathbb{N}\}$. Prove that L can be decided with space $O(\log\log n)$. Use this result to prove that $\mathrm{REG} \subsetneq \textsc{Dspace}(O(\log\log n))$.

Exercise 3

Let $\mathrm{PAL} = \{w \in \Sigma^* : w = \overleftarrow{w}\}$, where $\overleftarrow{w_0 \dots w_{n-1}} = w_{n-1}\dots w_0$, be the language of palindromes over a fixed alphabet Σ.

(a) Prove that $\mathrm{PAL} \in \textsc{Logspace}$ and $\mathrm{PAL} \in \textsc{Dtime}(O(n))$, and specify the respective time- and space-bounds of your algorithms.

(b) Prove that a Turing machine that is only allowed to move the head on the input tape to the right cannot decide the language PAL with logarithmic space.

Exercise 4

Prove, using the Gap Theorem, that there exists a computable function f for which $\textsc{Dspace}(f) = \textsc{Dtime}(f)$.