Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, K. Dannert

WS 2019/20

Algorithmic Model Theory — Assignment 5

Due: Tuesday, 12 November, 10:30

Exercise 1

5 Points

In the definition of SO-HORN, the clauses $\beta_1 \wedge \cdots \wedge \beta_m \to H$ are Horn-clauses only with respect to $R_1, \ldots, R_m$, since the β_i can be arbitrary FO(τ)-formulae (without R_i). We now consider a restriction SO-HORN* of SO-HORN, where the β_i have to be atomic FO-fromulae. Prove or disprove that SO-HORN* captures PTIME over ordered structures $(\mathfrak{A}, <)$.

Exercise 2

8 Points

(a) Describe the meaning of the LFP-sentence

$$\psi := \forall y \exists z Fyz \wedge \forall y[\text{lfp } Ry.\ \forall x(Fxy \to Rx)](y)$$

in your own words and explain why it is an infinity axiom.

(b) Describe the meaning of the LFP-sentence

$$\forall x \forall y(Exy \to \neg[\text{lfp } Pz.\ z = x \vee (\exists u \exists v((Ezu \vee Euz) \wedge (Euv \vee Evu) \wedge Pv))](y)).$$

Exercise 3

12 Points

(a) Let E be a binary relation symbol and P a unary relation symbol. Give an LFP($\{E, P\}$)-formula expressing the following properties:

(i) There is an element from which no E-path contains a node in P.

(ii) There is an element from which there is an E-path containing infinitely many nodes in P.

(b) Let GEN $= (A, f, S, u)$, where $f : A \times A \to A$ is a binary function, S is a unary predicate and u is a constant. Give an LFP-fromula φ such that GEN $\models \varphi$ if and only if u is in the closure of S under f.

Exercise 4

5 Points

Let $\varphi(x) = [\text{lfp } Ry.\ \exists x_1 \ldots \exists x_m \psi(R, y, x_1, \ldots, x_m)](x)$ with ψ quantifier-free. Prove that for all structures $\mathfrak{A}$ (finite or infinite) and all $a \in A$, $\mathfrak{A} \models \varphi(a)$ if and only if $a \in R^\omega$, i.e. the fixed point is reached at (or before) the first limit ordinal.