Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
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Algorithmic Model Theory — Assignment 3

Due: Tuesday, 29 October, 10:30

Exercise 1 15 Points

Prove that for the following classes, the satisfiability problem is decidable.

(i) The ($\exists^*$)-class, containing all FO formulae whose quantifier prefix in prenex normal form only contains existential quantifiers. Note that relations and functions of any arity are allowed.

Hint: Transform a given formula into a formula whose atoms are all of the form $Rx_1 \dots x_k$, $x_1 = x_2$ or $fx_1 \dots x_l = x_j$, where all x_i are variables.

(ii) The class of monadic formulae without equality, i.e. of fomulae without equality statements whose vocabulary contains only unary relation symbols and unary function symbols.

Hint: Construct a formula with at most n unary relation symbols and no function symbols that is satisfiable over the same universes as the original formula.

(iii) The class FO^+, consisting of all FO formulae that do not contain negation.

Exercise 2 15 Points

Show that the following classes of FO-sentences, where R is a binary relation symbol and f is a unary function symbol, contain infinity axioms.

(i) $\exists x \forall y \forall z \varphi(x, y, z)$, $\varphi \in \mathrm{FO}(\{f\})$ quantifier-free.

(ii) $\forall x \exists y \forall z \varphi(x, y, z)$, $\varphi \in \mathrm{FO}(\{R, f\})$ quantifier-free and withut equality.

(iii) $\forall x \exists y \varphi(x, y)$, $\varphi \in \mathrm{FO}(\{f\})$ quantifier-free.

(iv) The two variable fragment FO^2 extended by the counting quantifiers $\exists^{\leq n}$ for every $n \in \mathbb{N}$, where $\exists^{\leq n} x \varphi$ expresses that there are no more than n elements x that satisfy φ.