Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
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WS 2019/20

Algorithmic Model Theory — Assignment 2

Due: Tuesday, 22 October, 10:30

Exercise 1 5 Points

Let X be the class of all relational FO-formulas φ in prenex normal form

$$\varphi = \exists x_1 \cdots \exists x_r \, \forall y_1 \cdots \forall y_s \, \eta(\bar{x}, \bar{y}),$$

where η is quantifier-free.

We saw that X has the finite model property (see Assignment 1, Exercise 2). Hence, $\mathrm{Sat}(X)$, $\text{Fin-Sat}(X)$, and $\text{Non-Sat}(X)$ are decidable. Prove or disprove that $\mathrm{Val}(X)$ is decidable.

Exercise 2 10 Points

(a) Let X denote the set of all relational FO-formulas φ with binary relation symbols only and in prenex normal form

$$\varphi = \forall x \forall y \forall z \exists v \, \eta(x, y, z, v),$$

where η is quantifier-free.

Show that X is a conservative reduction class.

Hint: Use the same technique as in the reduction from the domino problem to the KMW-class ($\forall\exists\forall$), but use a binary relation to describe the successor function.

(b) We know from the lecture that the class $\mathcal{F}$, consisting of all sentences $\forall x \varphi$ where φ is quantifier free and has a vocabulary of only unary function symbols, is a conservative reduction class. Show that $\mathcal{F}_2 \subseteq \mathcal{F}$, consisting of sentences in $\mathcal{F}$ that contain just two unary functions, is also a conservative reduction class.

Hint: Transform sentences $\forall x \varphi$ with unary function symbols $f_1, \ldots, f_m$ into sentences $\forall x \tilde{\varphi} := \forall x \varphi[x/hx, f_i/hg^i]$ where h, g are fresh unary function symbols.

Exercise 3 15 Points

Which of the following subclasses of CORNER-DOMINO are r.e. and which are co-r.e.? In each case prove your answer.

(i) CORNER-DOMINO $= \{(\mathcal{D}, D_0)$: there exists a tiling of $\mathbb{N}\times\mathbb{N}$ by $\mathcal{D}$ with origin constraint $D_0\}$

(ii) CDOMINO-PER $= \{(\mathcal{D}, D_0)$: there exists a periodic tiling of $\mathbb{N}\times\mathbb{N}$ by $\mathcal{D}$ with origin constraint $D_0\}$

(iii) CDOMINO-NPER $= \{(\mathcal{D}, D_0)$: there exists a non-periodic tiling of $\mathbb{N} \times \mathbb{N}$ by $\mathcal{D}$ with origin constraint D_0 but no periodic one$\}$

(iv) CDOMINO-UNIQUE $= \{(\mathcal{D}, D_0)$: there exists a unique tiling of $\mathbb{N} \times \mathbb{N}$ by $\mathcal{D}$ with origin constraint $D_0\}$