Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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WS 2019/20

Algorithmic Model Theory — Assignment 1

Due: Tuesday, 15 October, 10:30

Note:
- You may work on the exercises in groups of up to three students.
- Hand in your solutions at the end of the lecture or put them into the box at the institute. Starting from sheet two, you may also hand them in at the beginning of the exercise class.

Exercise 1 4 + 5 + 2 Points

(a) Show that any two disjoint co-recursively enumerable languages $A, B \subseteq \Sigma^*$ are recursively separable, i.e. there exists a decidable set $C \subseteq \Sigma^*$ such that $A \subseteq C$ and $B \cap C = \emptyset$.

(b) Given a recursively enumerable language L, let $\text{code}(L) = \{\rho(M) \;:\; L(M) = L\}$. Show that if L_1 and L_2 are recursively enumerable languages and $L_1 \neq L_2$, then $\text{code}(L_1)$ is recursively inseparable from $\text{code}(L_2)$.

Hint: Use a reduction from a suitable pair of recursively inseparable sets.

(c) Prove or disprove that every pair of undecidable languages $A, B \subseteq \Sigma^*$ with $A \cap B = \emptyset$ is recursively inseparable.

Exercise 2 5 Points

Let X be the set of *relational* FO-sentences of the form $\exists x_1 \dots \exists x_r \forall y_1 \dots \forall y_s \, \varphi$ where $r, s \in \mathbb{N}$ and φ is quantifier-free. Show that $\text{Sat}(X)$ is decidable.

Hint: Show that each satisfiable sentence in X has a model with at most r elements.

Exercise 3 2 + 4 + 3 + 5 Points

Prove or disprove that the following decision problems are recursively enumerable and/or co-recursively enumerable. You may use that the validity problem and the finite satisfiability problem (i.e. deciding wether a formula has a finite model) for first-order logic are undecidable.

(a) $\text{NO-COMMON-MOD}_5 = \{\varphi \in \text{FO} : \text{for all } \psi, |\psi| = 5, \varphi \text{ has no common models with } \psi\}$

(b) $\text{ONLY-EVEN-MOD} = \{\varphi \in \text{FO} : \text{all finite models of } \varphi \text{ have even cardinality}\}$

(c) $\text{ALL-SHORT-EQV} = \{\varphi \in \text{FO} : \text{for all } \psi, |\psi| < |\varphi| \text{ it holds } \varphi \equiv \psi\}$

(d) $\text{ONE-SHORT-EQV} = \{\varphi \in \text{FO} : \text{there is } \psi, |\psi| < |\varphi| \text{ such that } \varphi \equiv \psi\}$.