

Algorithmic Model Theory — Assignment 12

Due: Monday, 27 January, 12:00

Exercise 1

Calculate the asymptotic probabilities of the following graph properties with respect to the uniform distribution on the class \mathcal{G} of undirected graphs:

- i) $\mathcal{K}_1 = \{G \in \mathcal{G} : G \text{ has no isolated node}\}$
- ii) $\mathcal{K}_2 = \{G \in \mathcal{G} : G \text{ is bipartite}\}$
- iii) $\mathcal{K}_3 = \{G \in \mathcal{G} : G \text{ is a tree}\}$
- iv) $\mathcal{K}_4 = \{G \in \mathcal{G} : G = (V, E) \text{ contains a clique of size } \geq \log(|V|)\}$

Exercise 2

Prove or disprove that the following logics have the zero-one law with respect to the uniform probability distribution on the respective classes. ($[n] := \{1, 2, \dots, n\}$)

- i) FO over the class of finite linear orders $\text{Lin} = \{([n], <) : n \in \mathbb{N}, < \text{ linear order on } [n]\}$
- ii) FO over the class of finite binary words $\text{W} = \{([n], <, P) : ([n], <) \in \text{Lin}, P \subseteq [n]\}$
- iii) FO over the class of bipartite graphs $\text{Bip} = \{([n] \times \{0, 1\}, E) : E \subseteq ([n] \times 0) \times ([n] \times 1)\}$
- iv) $\text{C}_{\infty, \omega}^{\omega}$ over the class of all graphs
- v) SO over the class of all graphs