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Algorithmic Model Theory — Assignment 5

Due: Monday, 25 November, 12:00

Exercise 1

For $n \geq 2$ we consider the directed path $\mathcal{P}_n$ of length n, i.e. the $\{E\}$-structure

$$\mathcal{P}_n = (\{0, \dots, n-1\}, \{(i, i+1) : 0 \leq i < n-1\}).$$

Construct for every $n \geq 2$ a sentence $\varphi_n \in \mathrm{FO}^2$ such that for every $\{E\}$-structure $\mathfrak{A}$ it holds $\mathfrak{A} \models \varphi_n$ if, and only if, $\mathfrak{A} \cong \mathcal{P}_n$.

Exercise 2

Recall the encoding of ordered structures presented in the lecture. Let $\tau = \{P, R\}$ be a signature consisting of a unary predicate P and a binary predicate R. Construct formulae $\beta_0(\bar{x})$ and $\beta_1(\bar{x})$ defining the $\bar{x}$-th symbol of the encoding of an ordered τ-structure.

Exercise 3

(a) Show that the following classes of (undirected) graphs are in NP by explicitly constructing Σ_1^1-sentences defining them.

  (i) The class of regular graphs (i.e. every node has the same number of neighbours),

  (ii) the class of Hamiltonian graphs, and

  (iii) the class of graphs that admit a perfect matching.

(b) Let $k \geq 1$. An (undirected) graph $G = (V, E)$ has connectivity k if $|G| > k$ and

  - for all $S \subseteq V, |S| < k$ the graph $G \setminus S$ is connected, and
  - there exists a set $S \subseteq V, |S| = k$ such that $G \setminus S$ is not connected.

  Construct a Σ_1^1-sentence defining the class of (undirected) graphs with connectivity k.