Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, F. Abu Zaid, W. Pakusa,
F. Reinhardt

WS 2013/14

Algorithmic Model Theory — Assignment 4

Due: Monday, 18 November, 12:00

Exercise 1

In this exercise we want to show that the model construction for FO^2-formulae from the lecture is optimal in the following sense: in general it does not suffice to take only two copies (instead of three) of the set P which consists of those atomic 1-types which are realised at least twice in $\mathfrak{A}$.

Find an example of a satisfiable FO^2-sentence $\varphi = \forall x \forall y \alpha \wedge \forall x \exists y \beta$ where α, β are quantifier-free such that:

- no model of φ contains a king (i.e. $K = \emptyset$) and
- for every model $\mathfrak{A}$ of φ there is no corresponding finite model over the universe $P \times \{0, 1\}$.

Exercise 2

Show that the class $[\exists^*\forall, (0), (1)]_=$ has the finite model property.

Hint: Consider the Skolem normal-form of such sentences φ and try to prune a possibly infinite model of φ by using the fact that in all terms that appear in φ the number of iterations of f is bounded.

Exercise 3

(a) Show that the problem whether a sentence of length n given in prenex normal form with q universal quantifiers has a model with at most s elements can be decided nondeterministically in time $p(s^q n)$ for some polynomial p.

(b) Conclude, using the arguments from Exercise 1 of Assignment 2, that $\mathrm{Sat}[\exists^*\forall^*, \mathrm{all}, (0)]_= \in$ NEXPTIME.

(c) Show that $\mathrm{Sat}[\exists^*\forall^*, \mathrm{all}, (0)]_=$ is even NEXPTIME-complete by proving the hardness via a reduction from $\mathrm{Domino}(\mathfrak{D}, 2^n)$ to $\mathrm{Sat}[\exists^2\forall^*, \mathrm{all}, (0)]_=$.

Hint: Use sentences of the form $\exists 0 \exists 1 \forall \bar{x} \forall \bar{y} \ldots (0 \neq 1 \wedge \varphi)$ where tuples $\bar{x} = x_0 \ldots x_{n-1}$ represent coordinates and φ describes a correct tiling using appropriate relations.