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Algorithmic Model Theory — Assignment 3

Due: Monday, 11 November, 12:00

Exercise 1

Construct a conservative reduction from $[\forall\exists\forall, (0,\omega), (0)]$ to $[\forall^2, (0,1), (1)]$.

Hint: Consider the Skolem normalform of a $\forall\exists\forall$-sentence to get rid of the $\exists$-quantifier. Encode multiple binary relations $R_1, \dots, R_n$ and the unary skolem function g by a single binary relation Q and a unary function f via the substitution $R_i xy \mapsto Qxf^i y$ for all $1 \leq i \leq n$ and $gx \mapsto f^{n+1}x$.

Exercise 2

Construct infinity axioms in the following classes

(i) $[\exists\forall^2, (0), (1)]_=$

(ii) $[\forall\exists\forall, (0,1), (1)]$

(iii) $[\forall\exists, (0), (1)]_=$

(iv) the two variable fragment FO^2 extended by the counting quantifiers $\exists^{\leq n}$ for every $n \in \mathbb{N}$, where $\exists^{\leq n} x\varphi$ expresses that there are no more than n elements x that satisfy φ.

Exercise 3

(a) For each of the following FO-formulae provide either an equivalent ML-formula or prove that no equivalent ML-formula exists.

Hint: Use the bisimulation invariance of ML for the non-existence proofs.

(i) $\varphi_1(x) := \forall y \exists z (Exy \lor Eyz)$;

(ii) $\varphi_2(x) := \forall y \exists z (\neg Exy \lor Eyz)$;

(iii) $\varphi_3(x) := \exists y \forall z (Eyx \land Eyz \land Pz)$.

(b) Show that it is undecidable whether a given FO-formula $\varphi(x)$ with only unary and binary relation symbols is equivalent to a ML-formula.