Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, F. Abu Zaid, W. Pakusa,
F. Reinhardt

WS 2013/14

Algorithmic Model Theory — Assignment 2

Due: Monday, 4 November, 12:00

Note: – You may work on the exercises in groups of up to three students.
– Hand in your solutions at the end of the lecture or put them into the box at the institute.

Exercise 1

Let X be the set of relational FO-sentences of the form $\exists x_1 \dots \exists x_r \forall y_1 \dots \forall y_s \, \varphi$ where $r, s \in \mathbb{N}$ and φ is quantifier-free. Show that $\text{Sat}(X)$ is decidable.

Hint: Show that each satisfiable sentence in X has a model with at most r elements.

Exercise 2

(a) Show that $[\forall^3\exists, (0, \omega), (0)]_=$ is a conservative reduction class.

Hint: Use the same technique as in reduction from the domino problem to the $\forall\exists\forall$-class, but use a binary relation to describe the successor function.

(b) Show that this even holds in the absence of equality, i.e. show that $[\forall^3\exists, (0, \omega), (0)]$ is a conservative reduction class.

Hint: Try to substitute equality by an appropriate congruence relation.

Exercise 3

Which of the following subclasses of CORNER-DOMINO are r.e. and which are co-r.e.? In each case prove your answer.

(i) CORNER-DOMINO = $\{(\mathcal{D}, D_0)$: there exists a tiling of $\mathbb{N}\times\mathbb{N}$ by $\mathcal{D}$ with origin constraint $D_0\}$

(ii) CDOMINO-PER = $\{(\mathcal{D}, D_0)$: there exists a periodic tiling of $\mathbb{N}\times\mathbb{N}$ by $\mathcal{D}$ with origin constraint $D_0\}$

(iii) CDOMINO-NPER = $\{(\mathcal{D}, D_0)$: there exists a non-periodic tiling of $\mathbb{N} \times \mathbb{N}$ by $\mathcal{D}$ with origin constraint D_0 but no periodic one$\}$

(iv) CDOMINO-UNIQUE = $\{(\mathcal{D}, D_0)$: there exists a unique tiling of $\mathbb{N} \times \mathbb{N}$ by $\mathcal{D}$ with origin constraint $D_0\}$