Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
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WS 2013/14

Algorithmic Model Theory — Assignment 1

Due: Monday, 28 October, 12:00

Note:
- You may work on the exercises in groups of up to three students.
- Hand in your solutions at the end of the lecture or put them into the box at the institute.

Exercise 1

(a) Show that any two disjoint co-recursively enumerable languages A and B are recursively separable, i.e. there exists a recursive set C such that $A \subseteq C$ and $B \cap C = \emptyset$.

(b) Given a recursively enumerable language L, let $\text{code}\, L = \{\rho(M) \;:\; L(M) = L\}$. Show that if L_1 and L_2 are recursively enumerable languages and $L_1 \subsetneq L_2$, then $\text{code}\, L_1$ is recursively inseparable from $\text{code}\, L_2$.

Hint: Use a reduction from a suitable pair of recursively inseparable sets.

(c) Prove or disprove that every pair of undecidable languages $A, B \subseteq \Sigma^*$ with $A \cap B = \emptyset$ is recursively inseparable.

Exercise 2

Prove or disprove that the following pairs of decision problems are recursively inseparable.

(a) $\text{A} = \{\rho(M) : \text{there is no } w, |w| \leq 2^{|\rho(M)|} \text{ s.th. } w \in L(M)\}$
$\text{B} = \{\rho(M) : \text{there is } w, |w| \leq 2^{|\rho(M)|} \text{ s.th. } M \text{ halts on } w \text{ within at most } 2^{|\rho(M)|} \text{ steps}\}$.

(b) $\text{EQ} = \{\rho(M)\sharp\rho(M') \;:\; L(M) = L(M')\}$
$\text{NEQ} = \{\rho(M)\sharp\rho(M') \;:\; (L(M) \setminus L(M')) \cup (L(M') \setminus L(M)) \neq \emptyset\}$.

Exercise 3

Prove or disprove (for example, by using Trakhtenbrot's Theorem) that the following decision problems are recursively enumerable and/or co-recursively enumerable.

(a) $\text{EVEN-SAT} = \{\varphi \in \text{FO} \;:\; \text{all finite models of } \varphi \text{ have even cardinality}\}$

(b) $\text{ALL-SHORT-EQV} = \{\varphi \in \text{FO} \;:\; \text{for all } \psi, |\psi| < |\varphi| \text{ it holds } \varphi \equiv \psi\}$

(c) $\text{ONE-SHORT-EQV} = \{\varphi \in \text{FO} \;:\; \text{there is } \psi, |\psi| < |\varphi| \text{ such that } \varphi \equiv \psi\}$.

Hint: Show that a decision algorithm for ONE-SHORT-EQV could be used to decide SAT(FO).