Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
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Algorithmic Model Theory — Assignment 13

Due: Monday, 30 January, 12:00

Exercise 1

For any directed graph $G = (V, E)$ we define its double graph $2G = (V', E')$ over the vertex set $V' = V \times \{0, 1\}$ with edge relation

$$E' := \{((u, i), (v, j)) \in V' \times V' : (u, v) \in E\}.$$

Let P_n denote the directed path of length n and let $\mathcal{K}$ be the class of all double directed paths, i.e. $\mathcal{K} := \{2P_n : n \geq 1\}$.

(a) Show that $\mathcal{K}$ does not admit an FP-definable linear order. *Hint:* Exercise 2, Sheet 11.

(b) Show that FP captures polynomial time on $\mathcal{K}$ by using the method of canonisation. Construct FP-interpretations (when necessary using equivalences) to show:
- P_n is interpretable in $2P_n$,
- $(C_n, 0)$ (an undirected circle of length n with a constant 0) is interpretable in P_n,
- $(2P_n, <)$ is interpretable in $(C_n, 0)$.

  Hint: Use the edge relation (in both directions) as the domain formula of your interpretation.

Exercise 2

In the lecture, the k-pebble bijection game was introduced which characterises $C^k_{\infty\omega}$-equivalence of structures.

(a) Modify the rules of the game to capture equivalence in $L^k_{\infty\omega}$ rather than $C^k_{\infty\omega}$.

Hint: Relax the requirement for Duplicator to choose a bijection.

(b) Use this game to show that the following classes of structures are undefinable in FP:
- The class of (undirected) graphs with an Eulerian cycle.

  Hint: Consider complete graphs.
- The class of (undirected) graphs with an Hamiltonian cycle.

  Hint: Consider complete bipartite graphs.

Exercise 3

Show that the CFI-query is decidable in polynomial time, i.e. show that given a CFI-graph $X_S(G)$ one can decide in polynomial time, whether S is even or odd.