

Algorithmic Model Theory — Assignment 3

Due: Monday, 7 November, 12:00

Exercise 1

In the lecture DOMINO was defined as the class of all domino systems \mathcal{D} which admit a tiling of $\mathbb{N} \times \mathbb{N}$. Show that DOMINO is co-r.e..

Hint: Construct a tree whose nodes are tilings of finite squares and use König's Lemma.

Exercise 2

Let $\Pi = \{\exists, \forall\}^{\leq k}$ be the set of all quantifier prefixes of length not larger than k for some fixed k and p a *finite* arity sequence (i.e. $\sum_{n \geq 1} p(n)$ is finite). Prove that $\text{Sat}([\Pi, p, (0)]_{=})$ is decidable.

Exercise 3

Construct infinity axioms in the classes

- (i) $[\exists \forall^2, (0), (1)]_{=}$
- (ii) $[\forall \exists \forall, (0, 1), (1)]$
- (iii) $[\forall \exists, (0), (1)]_{=}$