Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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Algorithmic Model Theory — Assignment 2

Due: Monday, 31 October, 12:00

Exercise 1

Prove or disprove that the following pairs of decision problems are recursively inseparable.

(a) A $= \{\rho(M) : \text{there is no } w, |w| \leq 2^{|\rho(M)|} \text{ s.th. } w \in L(M)\}$
B $= \{\rho(M) : \text{there is } w, |w| \leq 2^{|\rho(M)|} \text{ s.th. } M \text{ halts on } w \text{ within at most } 2^{|\rho(M)|} \text{ steps}\}$.

(b) EQ $= \{\rho(M)\sharp\rho(M') \,:\, L(M) = L(M')\}$
NEQ $= \{\rho(M)\sharp\rho(M') \,:\, (L(M) \setminus L(M')) \cup (L(M') \setminus L(M)) \neq \emptyset\}$.

Exercise 2

Prove or disprove that the following decision problems are recursively enumerable and/or co-recursively enumerable.

(a) EVEN-SAT $= \{\varphi \in \mathrm{FO} \,:\, \text{all finite models of } \varphi \text{ have even cardinality}\}$

(b) ALL-SHORT-EQV $= \{\varphi \in \mathrm{FO} \,:\, \text{for all } \psi, |\psi| \leq |\varphi| \text{ it holds } \varphi \equiv \psi\}$

(c) ONE-SHORT-EQV $= \{\varphi \in \mathrm{FO} \,:\, \text{there is } \psi, |\psi| \leq |\varphi| \text{ such that } \varphi \equiv \psi\}$.

Exercise 3*

Let Σ be a finite alphabet. A *Thue process over* Σ is a quotient of the semigroup $(\Sigma^+, \cdot)$. It is specified by a finite set of identities $I = \{(v_1, w_1), \ldots, (v_k, w_k)\} \subseteq (\Sigma^+ \times \Sigma^+)$ and defined as the quotient of $(\Sigma^+, \cdot)$ with respect to the smallest congruence relation $\sim_I$ containing

$$\{(x, y) \in \Sigma^+ \times \Sigma^+ \,:\, x = z_1 v_i z_2 \text{ and } y = z_1 w_i z_2 \text{ for some } (v_i, w_i) \in I \text{ and } z_1, z_2 \in \Sigma^\star\}.$$

The *word problem* for Thue processes over Σ is the following decision problem: Given a finite set of identities $I \subseteq \Sigma^+ \times \Sigma^+$ and two words $v, w \in \Sigma^\star$; decide whether $v \sim_I w$.

(a) Prove that the word problem for Thue processes is undecidable.

(b) Reduce the word problem for Thue processes to the validity problem for FO(τ), where $\tau = \{a_1, \ldots, a_n, f\}$ and $a_1, \ldots, a_n$ are constant symbols for the letters in Σ, and f is a binary function symbol.

Exercise 4

Prove that the set of relational first-order formulae without equality forms a reduction class.