Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, W. Pakusa, F. Reinhardt,
M. Voit

SS 2016

Algorithmic Model Theory — Assignment 2

Due: Friday, 29 April, 13:00

Exercise 1

Let X be the class of all relational FO-formulas φ in prenex normal form

$$\varphi = \exists x_1 \cdots \exists x_r \, \forall y_1 \cdots \forall y_s \, \eta(\bar{x}, \bar{y}),$$

where η is quantifier-free.

We saw that X has the finite model property (see Assignment 1, Exercise 2). Hence, $\mathrm{Sat}(X)$, $\text{Fin-Sat}(X)$, and $\text{Non-Sat}(X)$ are decidable. Prove or disprove that $\mathrm{Val}(X)$ is decidable.

Exercise 2

Prove that for every finite relational vocabulary τ and for every $k \geq 1$ the set $X = X[\tau, k]$ consisting of all FO-sentences of quantifier-rank at most k the satisfiability problem $\mathrm{Sat}(X)$ is decidable. Does X has the finite model property for all choices of τ and $k \geq 1$?

Exercise 3

(a) Let X denote the set of all relational FO-formulas φ with binary relation symbols only and in prenex normal form

$$\varphi = \forall x \forall y \forall z \exists v \, \eta(x, y, z, v),$$

where η is quantifier-free.

Show that X is a conservative reduction class.

Hint: Use the same technique as in the reduction from the domino problem to the KMW-class ($\forall\exists\forall$), but use a binary relation to describe the successor function.

(b) Show that this even holds for the subclass $Y \subseteq X$ of formulas in X without equality.

Hint: Try to substitute equality by an appropriate congruence relation.

Exercise 4

From the lecture we know that the following problems are pairwise recursively inseparable:

(i) DOMINO-NT $= \{\mathcal{D} : \text{there is no tiling of } \mathbb{N} \times \mathbb{N} \text{ by } \mathcal{D}\}$

(ii) DOMINO-PER $= \{\mathcal{D} : \text{there is a periodic tiling of } \mathbb{N} \times \mathbb{N} \text{ by } \mathcal{D}\}$

(iii) DOMINO-NPER $= \{\mathcal{D} : \text{there is an aperiodic tiling of } \mathbb{N} \times \mathbb{N} \text{ by } \mathcal{D}\text{, but no periodic one}\}$

Which of these problems are r.e. and which of them are co-r.e.? Justify your answer (by using the undecidability of the domino problem).