Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
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SS 2016

Algorithmic Model Theory — Assignment 1

Due: Friday, 22 April, 13:00

Note:
- You may work on the exercises in groups of up to three students.
- Hand in your solutions at the end of the lecture or put them into the box at the institute.

Exercise 1

(a) Show that any two disjoint co-recursively enumerable languages $A, B \subseteq \Sigma^*$ are recursively separable, i.e. there exists a recursive set $C \subseteq \Sigma^*$ such that $A \subseteq C$ and $B \cap C = \emptyset$.

(b) Given a recursively enumerable language L, let $\text{code}(L) = \{\rho(M) \,:\, L(M) = L\}$. Show that if L_1 and L_2 are recursively enumerable languages and $L_1 \neq L_2$, then $\text{code}(L_1)$ is recursively inseparable from $\text{code}(L_2)$.

Hint: Use a reduction from a suitable pair of recursively inseparable sets.

(c) Prove or disprove that every pair of undecidable languages $A, B \subseteq \Sigma^*$ with $A \cap B = \emptyset$ is recursively inseparable.

Exercise 2

Let X be the set of *relational* FO-sentences of the form $\exists x_1 \dots \exists x_r \forall y_1 \dots \forall y_s \, \varphi$ where $r, s \in \mathbb{N}$ and φ is quantifier-free. Show that $\text{Sat}(X)$ is decidable.

Hint: Show that each satisfiable sentence in X has a model with at most r elements.

Exercise 3

Prove or disprove (for example, by using Trakhtenbrot's Theorem) that the following decision problems are recursively enumerable and/or co-recursively enumerable.

(a) EVEN-SAT $= \{\varphi \in \text{FO} : \text{all finite models of } \varphi \text{ have even cardinality}\}$

(b) ALL-SHORT-EQV $= \{\varphi \in \text{FO} : \text{for all } \psi, |\psi| < |\varphi| \text{ it holds } \varphi \equiv \psi\}$

(c) ONE-SHORT-EQV $= \{\varphi \in \text{FO} : \text{there is } \psi, |\psi| < |\varphi| \text{ such that } \varphi \equiv \psi\}$.

Hint: Show that a decision algorithm for ONE-SHORT-EQV could be used to decide SAT(FO).