Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
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SS 2008

Algorithmic Model Theory — Assignment 2

Due: Tuesday, 29 April, 12:00

Exercise 1

Show that both Sat(X) and Finsat(X) are decidable in polynomial time if $X \subseteq \mathrm{FO}(\tau)$ is a class of sentences satisfying the following conditions: (a) τ is finite and relational, (b) all sentences in X are in prenex normal form, and (c) the number of variables of all $\psi \in X$ is bounded by a constant (i.e., there is a fixed $k \in \mathbb{N}$ such that for each $\psi \in X$ the length of the quantifier prefix of ψ is at most k).

Exercise 2

Construct, for each $n \in \mathbb{N}$, an FO^2-sentence ψ_n whose length is polynomial in n and whose models necessarily contain a $(2^n \times 2^n)$-grid.

An $(s \times t)$-grid is a graph over the set of vertices $\{0, \dots, s-1\} \times \{0, \dots, t-1\}$ with two edge relations $H = \{\langle (i,j), (i+1,j) \rangle : i < s-1, j < t\}$ and $V = \{\langle (i,j), (i,j+1) \rangle : i < s, j < t-1\}$.
Hint: The coordinates of the vertices of the grid have a binary representation of length n that can be described with unary relations. Using these, define appropriate successor relations.

Exercise 3

We consider a domino system $\mathfrak{D}$ given by (D, H, V) where D is a finite set of dominoes and the binary relations H and V contain those pairs of dominoes that may be placed side by side and on top of each other, respectively.

(a) Construct, for each domino system $\mathfrak{D} = (D, H, V)$, a set of propositional formulae $\Phi(\mathfrak{D})$ such that $\Phi(\mathfrak{D})$ is satisfiable if and only if $\mathfrak{D}$ admits a tiling of $\mathbb{Z} \times \mathbb{Z}$.

(b) Let $\mathfrak{D}$ be a domino system. Show that $\mathfrak{D}$ admits a tiling of $\mathbb{Z} \times \mathbb{Z}$ if and only if $\mathfrak{D}$ admits a tiling of arbitrarily large finite squares.