On logic in complexity theory and two different ways to do maths

Benedikt Pago Logic Mentoring Workshop 2023, Warsaw

Mathematical Foundations of Computer Science - RWTH Aachen University





Birds and Frogs

AMS Einstein Lecture 2008



Freeman Dyson

Birds and frogs in mathematics



Birds and frogs in mathematics



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- Logics are symmetry-invariant computation models.
- A logic \mathcal{L} captures a complexity class \mathcal{C} if the evaluation problem for \mathcal{L} -sentences is in \mathcal{C} and if every class of finite structures decidable in \mathcal{C} is definable by an \mathcal{L} -sentence.
- Central question in finite model theory: Does there exist a logic that captures PTIME?
- If not, then $\mathsf{P} \neq \mathsf{NP}$ (because existential second-order logic captures NP).
- Proving lower bounds against logics is easier than for general algorithms.

Landscape of logics contained in PTIME



ixed-point logic with counting (FPC ा Least fixed-point logic (LFP) ा

First-order logic (FO)

Choiceless Polynomial Time

CPT is a "functional programming language" with hereditarily finite sets as data structures. A CPT-program is built out of set-theoretic terms, such as:

- Pair(*a*, *b*) := {*a*, *b*}.
- $Union(a) := \bigcup a$.
- Comprehension: $\{t : x \in a : \varphi\} := \{t(x) \mid x \in a, \mathfrak{A} \models \varphi(x)\}.$
- Iteration: Terms can be iteratively applied to their own output.

Limitations of CPT:

- Length of iterations and size of constructed sets *polynomially bounded* in input size $|\mathfrak{A}|$.
- The computed sets are symmetric under the automorphisms of \mathfrak{A} .
- Goal: Prove inexpressibility results for CPT.

Symmetry-invariance of hereditarily finite sets



- $\{\{a, b, c\}, \{d, e, f\}\}$ is symmetric.
- {*a*, *d*} is *not* symmetric.

Lower bounds for CPT via analysis of highly symmetric structures



The 4-dimensional hypercube.

Theorem (P., CSL 2021) No CPT-program can define an ordered partition of the n-dimensional hypercube into parts of at most logarithmic size.

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Another approach: Lower bounds for CPT via propositional proof complexity



Theorem (P., CSL 2023) If CPT can distinguish all pairs of non-isomorphic graphs in a graph class K, then so can the degree-3 extended polynomial calculus (EPC₃).

Consequence: Lower bounds for the graph isomorphism problem in EPC₃ transfer to CPT.

Part II: PhD impressions

- Check out advertised positions on mailing lists such as **DMANET** or the **finite-model-theory list**.
- There are generally more open positions than applicants.
- Choose a **topic** you personally find **exciting**.
- Other aspects to consider: is the supervisor active in research, how big is the group, how much teaching and other tasks does the job involve?

Pros and Cons of being a theoretical researcher

Pros	Cons
self-dependance	self-dependance
freedom	less money than in industry
extremely challenging	lack of relevance
varying topics	
teaching	
interesting people	

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