Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, D. Fischer, T. Ganzow,
B. Puchala

WS 09/10

Complexity Theory and Quantum Computing — Assignment 10

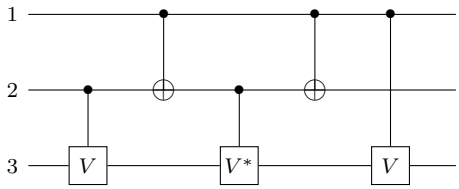
Due: Monday, January 18, 12:00

Exercise 1

Prove that any unitary matrix $U \in \mathbb{C}^{n\times n}$ has a square root, that means, there is a unitary matrix V such that $V \cdot V = U$.
Hint: Use the fact that U is diagonalizable via a basis-transformation with an orthonormal basis of $\mathbb{C}^n$, consisting of eigenvectors of U.

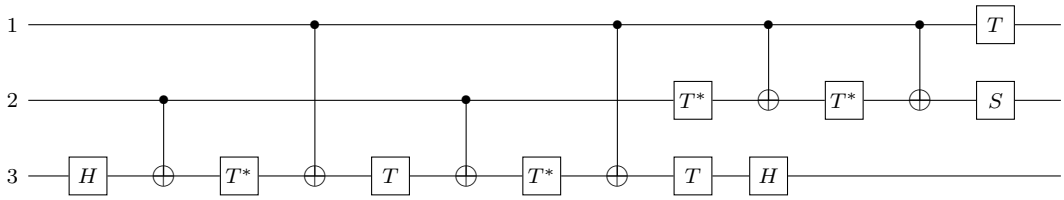
Exercise 2

Let $U \in \mathbb{C}^{n\times n}$ be a unitary matrix and let $V \in \mathbb{C}^{n\times n}$ be a square root of U. Prove that the following quantum gate array over $\{C{-}\mathrm{NOT}, C{-}V, C{-}V^*\}$ computes U.



Exercise 3

Consider the following quantum gate array,



where the gates S, T and H are given by the following unitary matrices.

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}, H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Prove that this QGA computes the Toffoli-gate.
Hint: Calculate appropriate products of the matrices S, T, H and $C{-}\mathrm{NOT}$ and prove that the given QGA maps any state $|0yz\rangle$ to $|0yz\rangle$, any state $|10z\rangle$ to $|10z\rangle$ and any state $|11z\rangle$ to $|1\rangle \otimes |1\rangle \otimes |1 \oplus z\rangle$.