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WS 09/10

Complexity Theory and Quantum Computing — Assignment 6

Due: Monday, December 07, 12:00

Exercise 1

A problem A is called polynomially Turing reducible to a problem B (denoted $A \leq_P^T B$) if $A \in P^B$, i.e., there is a deterministic polynomially time bounded Turing machine with oracle B which decides A. A complexity class $\mathcal{C}$ is called closed under $\leq_P^T$ if for each $B \in \mathcal{C}$ and for each problem A with $A \leq_P^T B$ we also have $A \in \mathcal{C}$. Which of the following complexity classes are closed under $\leq_P^T$?

$$\mathrm{P}, \quad \mathrm{NP}, \quad \Sigma_k^p, \quad \Delta_k^p, \quad \mathrm{PH}, \quad \textsc{Pspace}$$

Exercise 2

Prove the following facts.

(a) The set of Boolean formulas φ for which there is no equivalent Boolean formula ψ with $|\psi| < |\varphi|$ is in Π_2^p.

(b) UNIQUE-TSP $\in \Delta_2^p$, where UNIQUE-TSP is the set of distance matrices for which there is a *unique* optimal tour (see assignment 1, exercise 3).

Exercise 3

If $G = (V, E)$ is a finite graph, $\mathrm{VC}_{\mathrm{path}}(G)$ is the size of the largest set $X \subseteq V$ which is shattered by paths in G, i.e., for each $S \subseteq X$ there is a path in G which contains all vertices of S but no vertex of $X \setminus S$. PATH VC DIMENSION is the following problem: Given a finite graph G and a natural number k, is $\mathrm{VC}_{\mathrm{path}}(G) \geq k$? Prove that PATH VC DIMENSION is in Σ_3^p.

Exercise 4

We say that an alternating Turing machine M makes at most $A(n)$ alternations if, for each input x of length n, on each computation path of M on x, the machine changes at most $A(n)$ times from an existential state to a universal state or vice versa. For $k \in \mathbb{N}$ let $\mathcal{C}_k := \textsc{Atime-Alt}(\bigcup_{d \in \mathbb{N}} n^d, k)$ be the class of languages which can be decided by a polynomially time bounded alternating Turing machine which makes at most k alternations.

(a) Explain the relationship between the classes $\mathcal{C}_k$ and the classes of the polynomial hierarchy.

(b) Assume that there is a complete problem for the class $\mathcal{C}_k$ for some k. Which implications does this have for the polynomial hierarchy?